# 3D electromagnetic modeling using atomic survey decomposition

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## SUMMARY

3D rigorous EM modeling is computationally challenging. The computational complexity arises from the physical size of the models and the large number of cells needed to adequately solve Maxwell's equations, the large number of transmitters, especially in airborne data, and the number of frequencies or times at which data are collected. In this paper we present a methodology whereby each datum, corresponding to a specific transmitter and receiver and a particular time channel has its own mesh for forward modelling. If the source is extended, as it is with long wires or large loops on the surface, we subdivide these sources into small line sources (linelets) for a grounded survey or small inductive sources (looplets) for an inductive source. In such circumstances the final datum is obtained by summing the responses from the individual linelets or looplets. We view the procedure as an advanced survey decomposition with the essential computation being reduced to finding a smallest feasible mesh and associated modelling parameters that can achieve a desired accuracy. We refer to these basic structures as atomic blocks. A major benefit of this approach is that it is trivially parallelized for truly large scale problems since each self-contained atomic block can be run on a separate processor and therefore we can easily take advantage of the evolving large scale processing capability that is becoming a reality. Our framework is generic and we showcase its usefulness in time-domain EM (TEM) for two types of surveys, airborne and ground large loop. We show that with this advanced survey decomposition that the forward responses and sensitivity can be efficiently computed without sacrifice of accuracy. Savings in the forward modelling thus transcribe to computational efficiencies in inversion. This framework can be easily extended to other types of EM survey modeling.

## INTRODUCTION

Electromagnetic forward and inverse modelings, particularly in high dimensions using techniques like finite element and finite difference, are usually computationally expensive, making rigorous and quantitative interpretations less usable. The computational costs can be attributed to three aspects: space complexity, time complexity and optimization complexity.

The primary concern of 3D modeling is the large number of cells needed to discretize the volume of interest and possibly a large number of sources. The space complexity becomes a prohibitive issue because of the notoriously poor scalability of computational costs versus the number of model cells. The number of cells can be reduced by using sophisticated meshes, for example a tetrahedral mesh (Schwarzbach and Haber, 2013), EM footprint (Cox et al., 2010), or by transforming the model to a low-dimensional subspace (Oldenburg et al., 1993), but the number of model parameters can still be unmanageable if the survey is really large. Alternatively, a large problem can be tackled by massive parallelization after fine-grained domain decomposition, but the benefit of adding more processors may be quickly marginalized due to the overhead of intercommunication (Newman and Alumbaugh, 1997; Commer et al., 2008).

The time complexity arises as the modeling in Maxwell's equations is usually simulated by either stepping in time or done spectrally with frequencies. EM diffusion evolves at time scales from about 0.01 ms to 1 s for most earth models, so appropriate time steps and/or discrete frequencies must be carefully chosen. One widely-adopted solution to this multi-scale problem in time is to adapt the step length to the scale of EM diffusion (Um et al., 2010; Oldenburg et al., 2013). In the frequency domain, it has been shown that the optimal set of frequencies can be found by the reduced Krylov subspace method (Zaslavsky et al., 2011). However, there is another type of over-computing that has not received sufficient attention. If many time channels from early to late are modeled together in one attempt, the step lengths required by the early times are unnecessarily small for the modeling at late times.

In this work we build upon our previous efforts (Yang et al., 2014) where we used the concept of local meshes in solving the airborne TEM inverse problem. Here we introduce a more sophisticated survey decomposition where every datum is obtained by summing contributions from highly specialized atomic building blocks. Each atomic block consists of modelling a small galvanic or inductive source at a particular time (or frequency). The atomic block consists of a local mesh and any associated parameters needed for the forward modelling. The final datum is a summation of simulations from the collection of atomic block. In airborne TEM, every sounding is modeled independently on its own local mesh. For a large ground loop, the transmitter is decomposed into small loops (looplets). The atomic block encapsulates the loop size, cell size, and parameters for time-stepping Maxwell's equations. These parameters depend upon the conductivity of the earth and the time at which the fields are to be calculated.

In the following we provide background about our TEM forward and inverse modelling approaches, present our atomic survey decomposition and then show its applicability to airborne and ground TEM problems.

## TEM MODELING ALGORITHMS

The basic forward and inverse algorithms we use are the same as Oldenburg et al. (2013). The modeling domain is discretized on a rectangular mesh and the differential operators in Maxwell's equations are approximated using a finite volume method on a staggered grid. The resulting large and sparse system of equations is solved with a direct solver (eg. Cholesky decomposition), with the advantage that once the Maxwell matrix is factorized, the forward responses and sensitivities, represented



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by many different right-hand side terms, can be quickly computed. The time derivative is approximated by the backward Euler (implicit stepping) method, which requires the time step to be much smaller than the scale of the observation time after transmitter current turn-off. For a given model, the Maxwell matrix will change if the time step changes, so in practice, if many time channels from early to late are desired, it is appropriate to adjust the step length every decade to match the time scale of the EM diffusion. In this way, only a few matrices need to be factorized for each modeling.

For the inverse problem, we are more interested in computing the sensitivity, particularly the explicit sensitivity. We have shown in Yang et al. (2014) that computing and storing the sensitivity matrices can be more efficient than using an implicit approach for many problems.

## SURVEY DECOMPOSITION

The ultimate goal of decomposing an EM survey is to promote “localized discretizations” to avoid the over-computing associated with solving a big problem that handles different scales from different sources, receivers and time channels. While an EM survey can be decomposed in many sensible ways, there are two keys issues to be addressed to gain efficiency: (1) sub-problems must target a small subset of the data, so its discretizations can be highly customized; (2) every sub-problem should be self-contained to minimumize intercommunication. Based on these two principles, we pre-construct atomic blocks in TEM, then use the solutions of the atomic blocks to build up different surveys.

An atomic problem involves a dipole transmitter (source) and a receiver making a point measurement at a particular TEM time channel. The transmitter and the receiver can be at arbitrary locations. Because the EM field diffuses, the mesh only needs to be locally refined near the source and receiver. This can result in a local mesh that is much smaller than a large global mesh covering the entire survey area with fine cells everywhere. The size of the finest cell is determined by the earliest time channel in a modeling, and the size of the modeling domain is tied to the latest time channel. By considering only one time channel in an atomic problem, the scale contrast between cells is minimized, yielding high efficiency of discretization; regardless of what other step lengths may be required by other time channels, one constant step length, which is usually about 1/10 to 1/20 of the measurement time, is all we need for the time stepping from zero to the atomic problem’s measurement time.

In practice, the conductivity model may vary from one location to another. We use an adaptive procedure to find the appropriate cell size and domain size. The program starts with a cheap initial guess, then refines and expands the mesh until the difference made between data evaluated with successive discretizations is negligible. Figure 1 shows an example of a structured mesh designed for such a typical atomic problem. Unstructured or semi-structured meshes can also be used. Figure 2 shows a time stepping scheme customized to time channel $t$, for which the step length $\delta t$ is one tenth of $t$; only one factorization of the Maxwell matrix is needed.

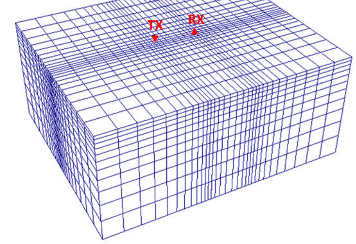


Figure 1: A local mesh designed for an atomic problem working for the source and receiver marked by the red triangles.

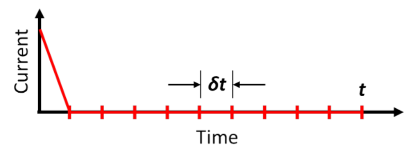


Figure 2: A time discretization with constant step length designed for an atomic problem working for time channel $t$.

Each atomic block is solved on an independent processor and works with a local mesh. There are two types of interactions between the demands at the global level and the computing at the local level. First, a conductivity model defined on a global mesh is passed to the local meshes; this possibly involves coordinates system transformation and can be implemented with intersecting volume-weighted averaging for the mesh conversion. After the modeling on local meshes, the computed forward responses and sensitivities are transformed back to the global system through rotation; for sensitivities, there is additional operation of interpolation that maps the sensitivities from the local mesh to the global mesh before rotation

$$\mathbf{J}_{gi} = \mathbf{J}_{li}\mathbf{V}_{li}^{-1}\mathbf{R}_i\mathbf{V}_g, \tag{1}$$

where the subscript $i$ represents the $i$th datum, $\mathbf{J}_g$ and $\mathbf{J}_l$ are the sensitivity on the global mesh and local mesh respectively, $\mathbf{R}$ is a 3D interpolation matrix, and $\mathbf{V}_g$ and $\mathbf{V}_l$ are diagonal matrices of cell volumes on the global mesh and local mesh.

We note that equation 1 does not imply that the sensitivity on the global mesh must be explicitly formed. In fact if an iterative method, which requires only $\mathbf{J}$ times a vector, is to be used in inversion, the vector can be passed to the local meshes and the matrix-vector multiplication can be carried out at local level in parallel before the resulting vectors are sent back to the global thread. Therefore, we only store $\mathbf{J}$ on local meshes.

With an atomic survey decomposition all computationally intensive processes in EM modeling, including evaluation of forward responses and development of a sensitivity matrix and its product with a vector, can be implemented on local meshes in parallel with very little intercommunication. Thus the computation time for the forward and inverse problems scales nearly linearly with the number of processors (see Yang et al. (2014)



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for an example of this using a simple local mesh). Next we discuss how the atomic survey decomposition can be used to model airborne and ground TEM surveys.

## AIRBORNE TEM

An airborne TEM survey consists of many individual soundings, each of which has one dipole transmitter and one receiver. Atomic blocks sharing the same source and receiver but different time channels can be assembled to model one individual sounding. The source and receiver are bundled and mostly coincident in airborne, so the variant of local mesh for airborne only has one local refinement at the sounding location. Figure 3 shows the space decomposition of an airborne survey with three soundings (illustrative only and not to scale).

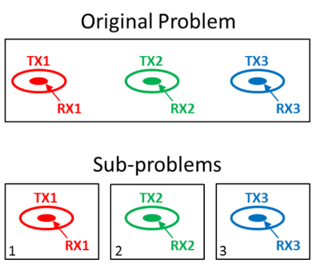


Figure 3: Airborne atomic survey decomposition in space

Since the bandwidth of an airborne survey is relatively narrow, the scale contrast between early and late times is moderate. Thus the decomposition in time can be optional if a simpler implementation is desired. We have published the results of a 3D airborne TEM inversion using only space decomposition (Yang et al., 2014). The saving was significant even without time decomposition. In that paper, the optimization complexity resulting from a large number of data was also greatly reduced by adaptive random down-sampling of the soundings.

## GROUND TEM

Except for the surveys using a small moving loop, the spatial layout of ground survey can be more complicated: (1) the transmitter loop is much larger than a magnetic dipole and can be arbitrarily shaped; (2) one transmitter may have many receivers with varying offsets. This is tackled by using superposition: the large transmitter loop is broken down to many small magnetic dipole sources, called "looplets"; then the looplets are paired with every receiver to form many atomic blocks; the simulated data are the sum of responses from all the looplets. Figure 4 uses a simple example (not to scale) to demonstrate how a large ground loop survey can be decomposed to many atomic blocks.

Most ground TEM measurements span a much greater time scale than in the airborne case, so decomposition in time is usually necessary. Although there seems a large number of atomic problems generated after the decomposition in space

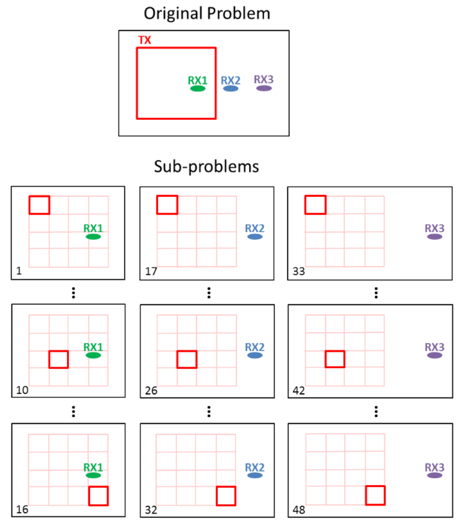


Figure 4: Ground atomic survey decomposition in space

and in time, there is potential of great saving thanks to the multi-scale nature of the EM diffusion. It is well known that EM induction data lose resolution rapidly as the time lapses, which means the large transmitter can be effectively represented by fewer looplets at late times; the receiver locations can also be down-sampled according to the data resolution at different times.

We use a synthetic modeling to show the effectiveness of survey decomposition in ground TEM. The model consists of a very conductive sphere buried in a resistive uniform half-space; the survey has one large transmitter loop of size 0.9 $\times$ 1.5 km (Figure 5). A receiver at (-900, 500) is chosen for the test. The global mesh has 169136 ($62\times62\times44$) cells with the smallest cell size 100 m. A complete forward modeling of this survey for time channels 0.1 $\sim$ 100 ms requires 4 different step lengths and 15 time steps per length (60 steps in total); The CPU time is 234 s on a 6-core computer in parallel mode; The peak memory usage is 11.4 GB.

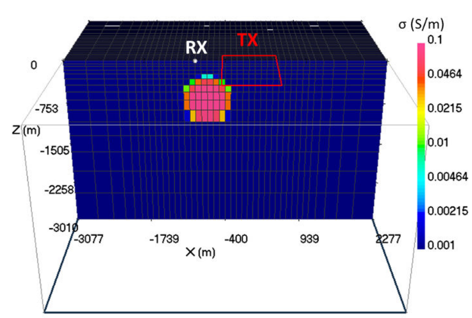


Figure 5: Synthetic model on global mesh with the padding zone removed

Next we compute this model using atomic blocks. At a particular time channel, we initialize $N$ looplets at random loca-



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tions inside the loop. Then $N$ atomic problems are solved for every looplet-receiver pair on $N$ local meshes. The final modeling results are the linear combination of the modeling results from the looplets with weights (looplet dipole moments) found by the areas of Voronoi cells. Because the magnetic field is smooth, a small $N$ is usually sufficient, especially at late times. Again, we use the idea of "refine until no difference" to adaptively find the minimum $N$ for a particular receiver at a particular time: we start with only one looplet; then gradually add more looplets until the difference made by adding additional looplets is below a tolerance; the Voronoi diagram is updated every time $N$ changes. Later time channels generally need a smaller $N$. Figure 6 shows the plan view of the transmitter loop (red lines), 7 looplet locations (black dots) required at 0.1 ms and the Voronoi partition of the loop.

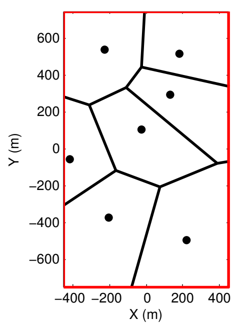


Figure 6: Partitioning the large transmitter loop with randomly located dipoles and associated Voronoi diagram

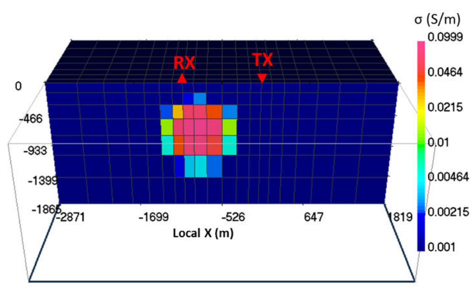


Figure 7: Synthetic model on a local mesh for a particular dipole-receiver pair at 0.1 ms

Figure 7 shows one of the 7 local meshes and local models used for the modeling at 0.1 ms (coordinates in local system after rotation and translate). This mesh serves one looplet-receiver pair and has only 3920 ($20\times14\times14$) cells; this is considerably fewer than the number of cells in the global mesh. Since only one time channel is modeled, there is only one step length needed and 15 time steps in total. Modeling on such a mesh takes 1.6 s on the same computer but using one core only and the memory usage is hardly noticeable. The total CPU time required by modeling the entire survey will depend on the number of processors available and this is where massive parallelization can come play and save time.

The modeled data of H field from the global mesh and atomic blocks are compared in Figure 8 and good agreement is observed. The strong components always have the best accuracy.

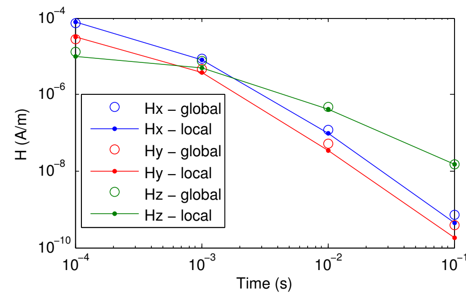


Figure 8: Comparison of H-field data between modeling on the global mesh and the atomic blocks' local meshes

We note that the computational costs will increase dramatically for the standard modeling on a global mesh if the survey becomes larger, but the modeling using atomic survey decomposition will scale much better because the EM problem is never solved on the global mesh.

## CONCLUSIONS

We examine the standard algorithm of EM modeling and identify excessive over-computing associated with space and time complexities. We therefore propose a new framework for EM induction modeling using atomic survey decomposition. Under this framework, any EM survey can be represented as a collection of atomic problems, which are self-contained subproblems that work with a localized mesh, a specific transmitter and receiver, and a specific time or frequency. The minimal dependence between the atomic problems allows implementation of massive parallelization available in cloud computing.

As an example, we show how atomic survey decomposition works in TEM. Two types of TEM surveys, airborne (moving dipole source) and ground (fixed large loop), are decomposed. We compare the modeling results and performances of survey decomposition to the standard method. High quality simulation results are obtaind at greatly reduced computational costs.

This framework is generic and can be adopted in other EM surveys for both forward modeling and inversion. It also opens up new opportunities of developing a further speedup for EM data modeling and we are exploring some of these. The niche for this algorithm is tied to large scale modelling and the availability of a large number of processors. When this is combined with practical speedups of the inversion using a stochastic approach such as that outlined in Yang et al. (2014) then very large EM inverse problems should be doable in practical times.

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**EDITED REFERENCES**

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