Cooperative Magnetic Inversion

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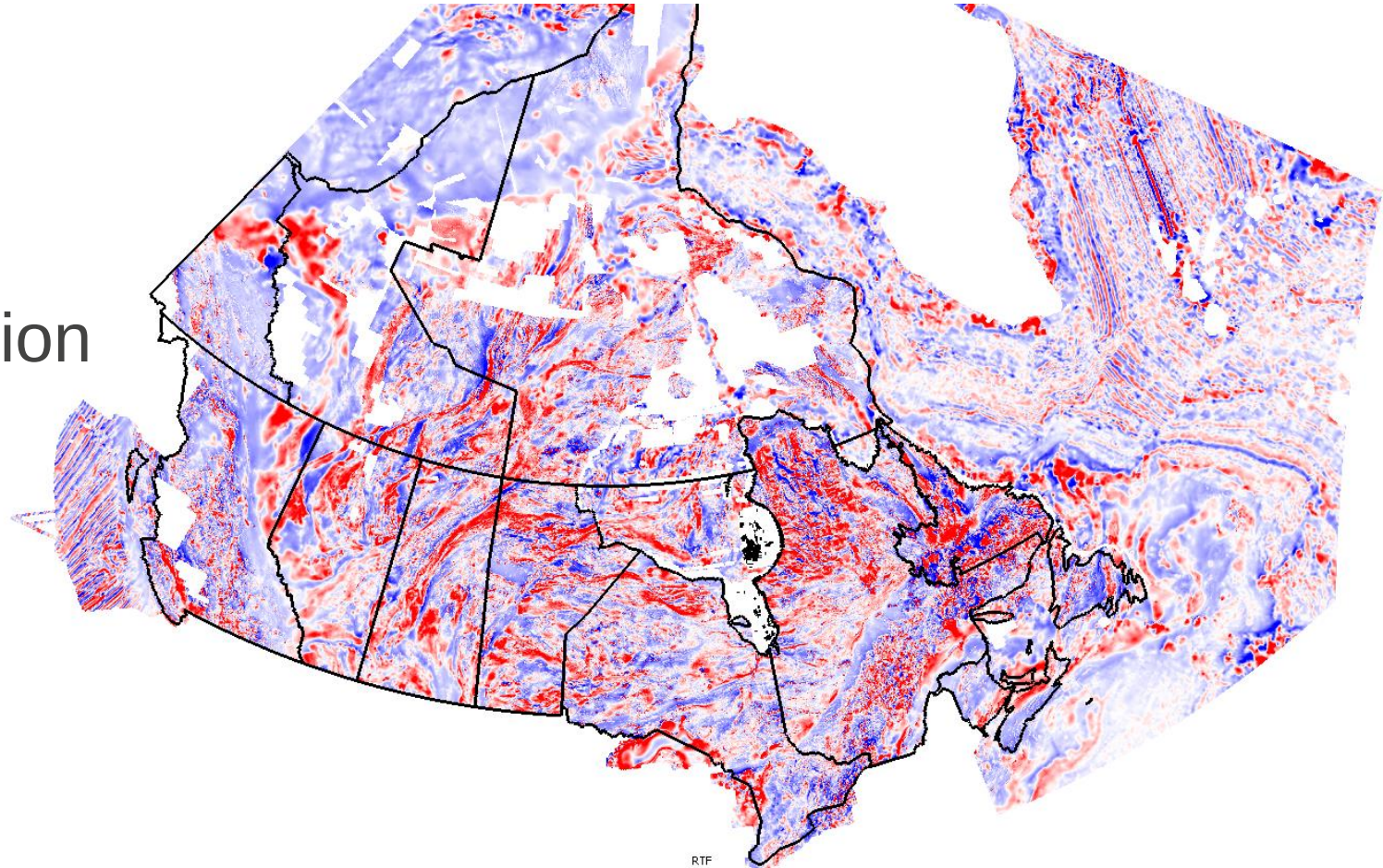
October 17th, 2016

Outline

- Review on magnetics
- Effect of remanence
- Review on algorithms
  - Amplitude Inversion
  - Magnetic Vector Inversion
- Cooperative strategy
- Ongoing research



Magnetic Data

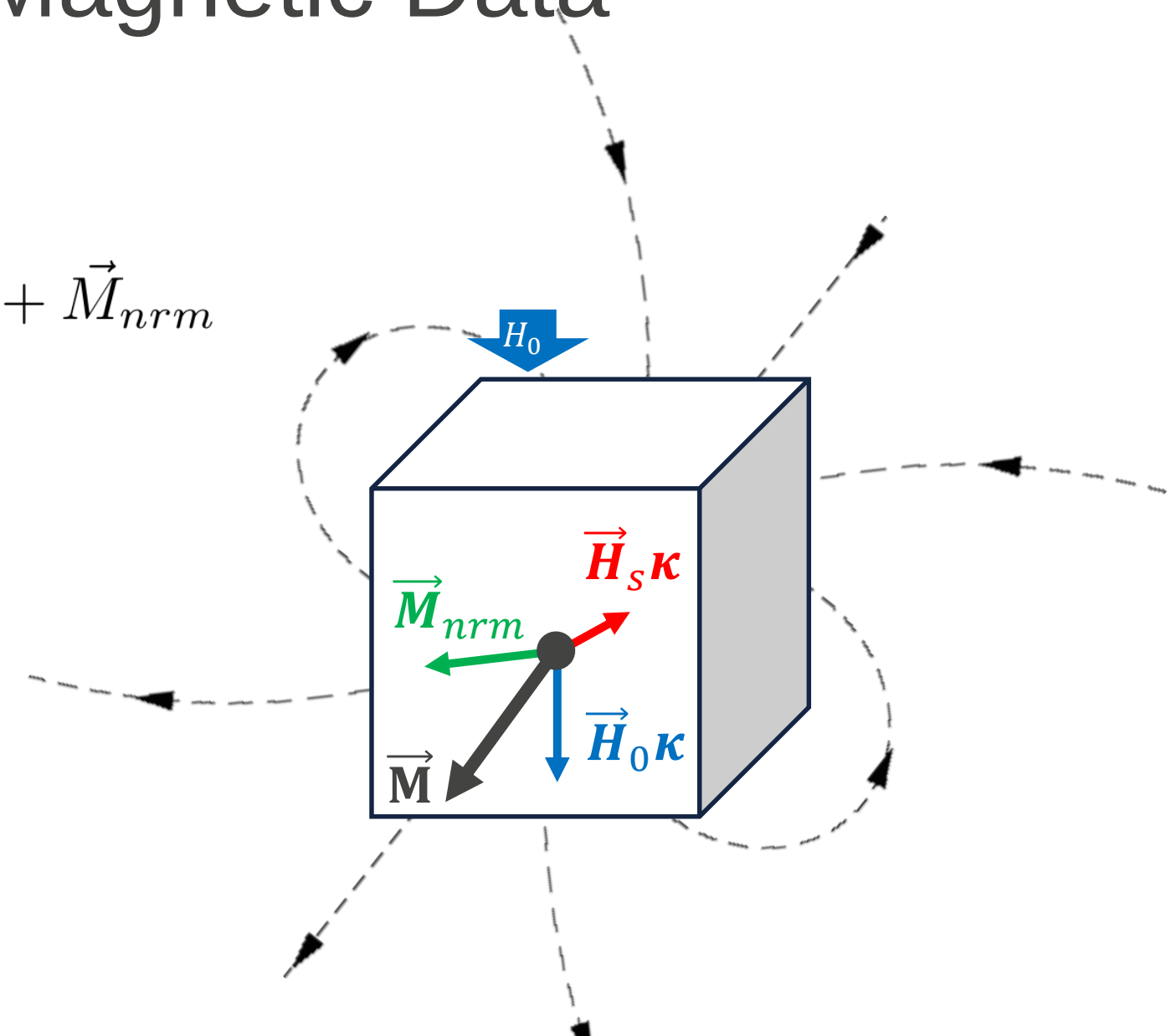
- Magnetization

$$\vec{M} = \kappa(\vec{H}_0 + \vec{H}_s) + \vec{M}_{nrm}$$

$\kappa\vec{H}_0 : induced$

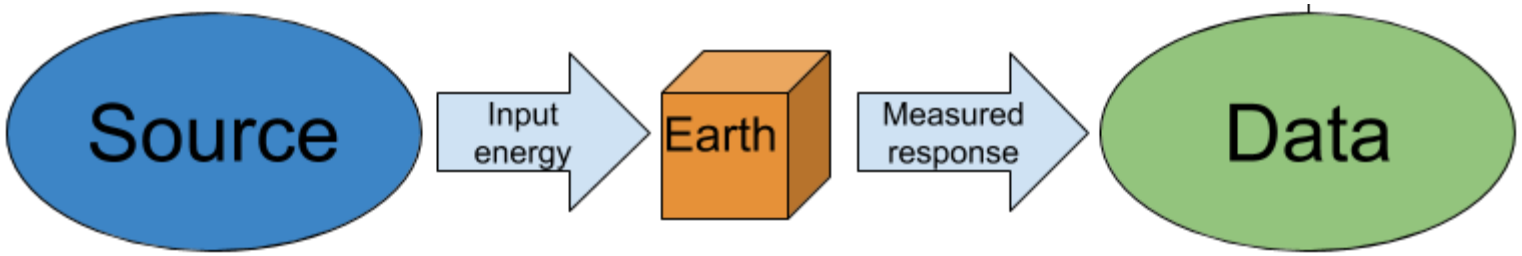
$\kappa\vec{H}_s : self - demag$

$\vec{M}_{NRM} : Remanence$

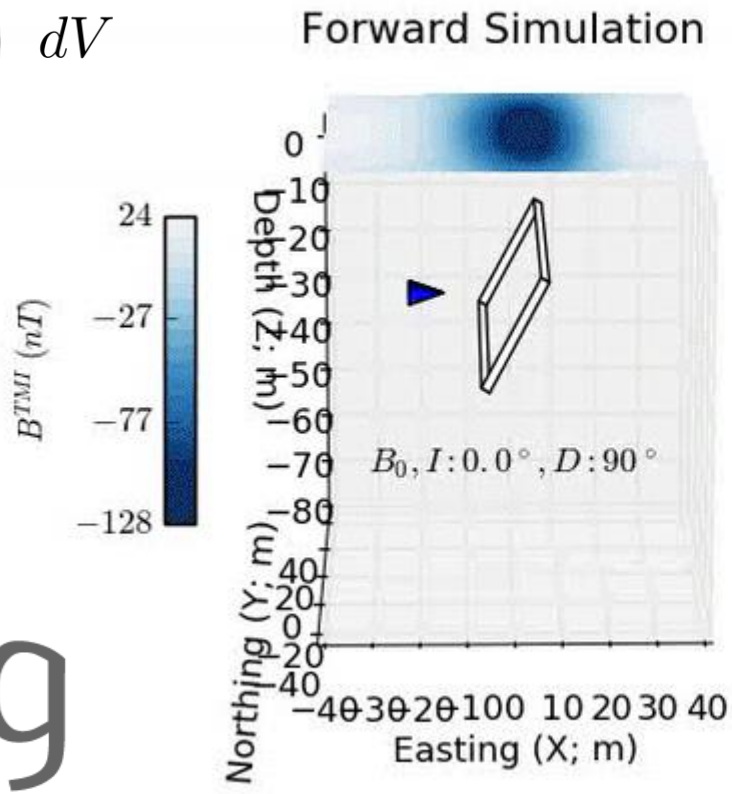


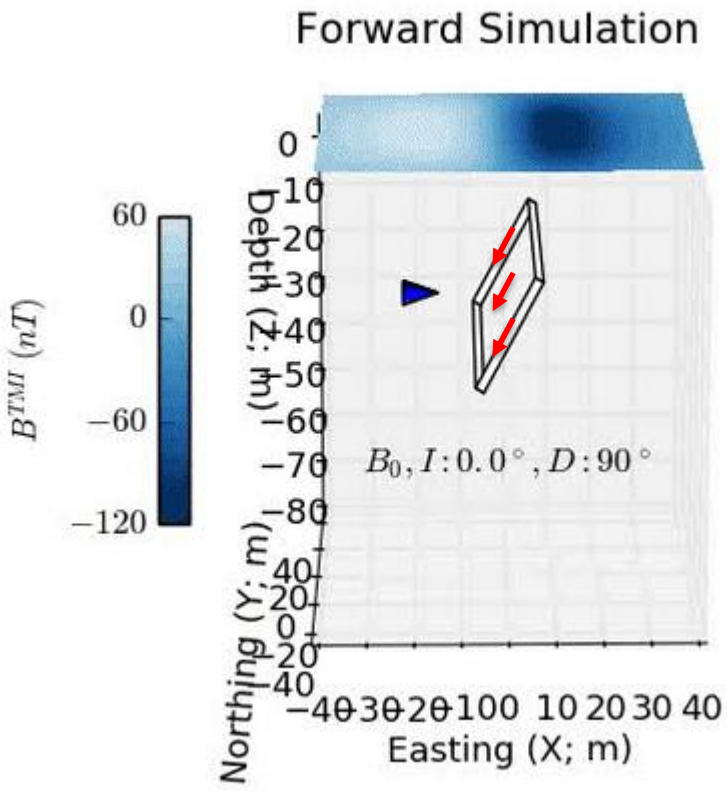
Magnetic Data

- Magnetic Data



$$\vec{b}(P) = \frac{\mu_0}{4\pi} \int_V \vec{M} \cdot \nabla\nabla \left(\frac{1}{r}\right) \, dV$$





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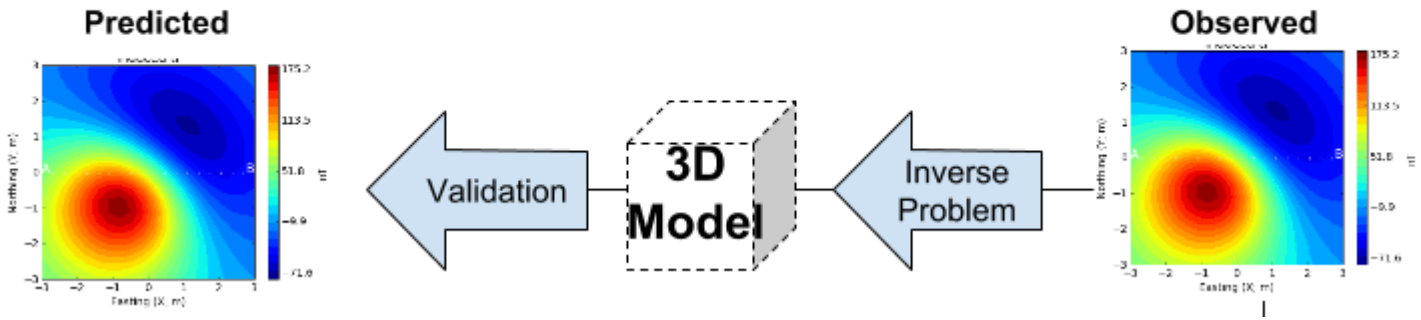
Magnetic Inversion

- Inverse Problem



This talk

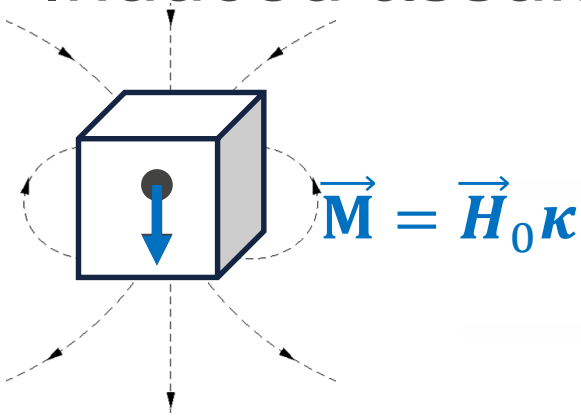
$$\min_{m} \boxed{\|\mathbf{F}[\mathbf{m}] - \mathbf{d}^{obs}\|_2^2} + \boxed{\beta\ \phi_m}$$

$$Data\ misfit + Regularization$$

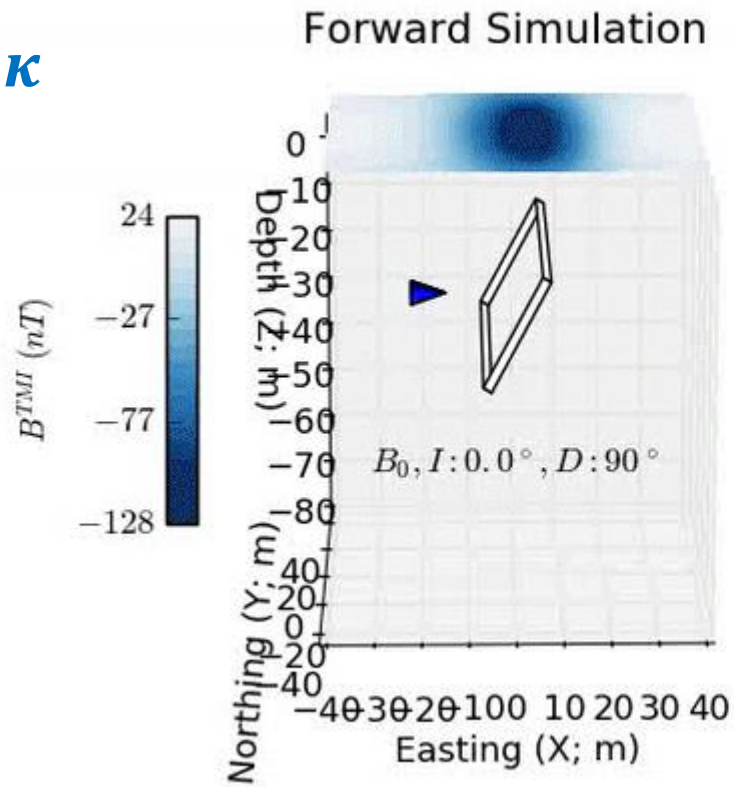
Next talk !! (4:10 pm)

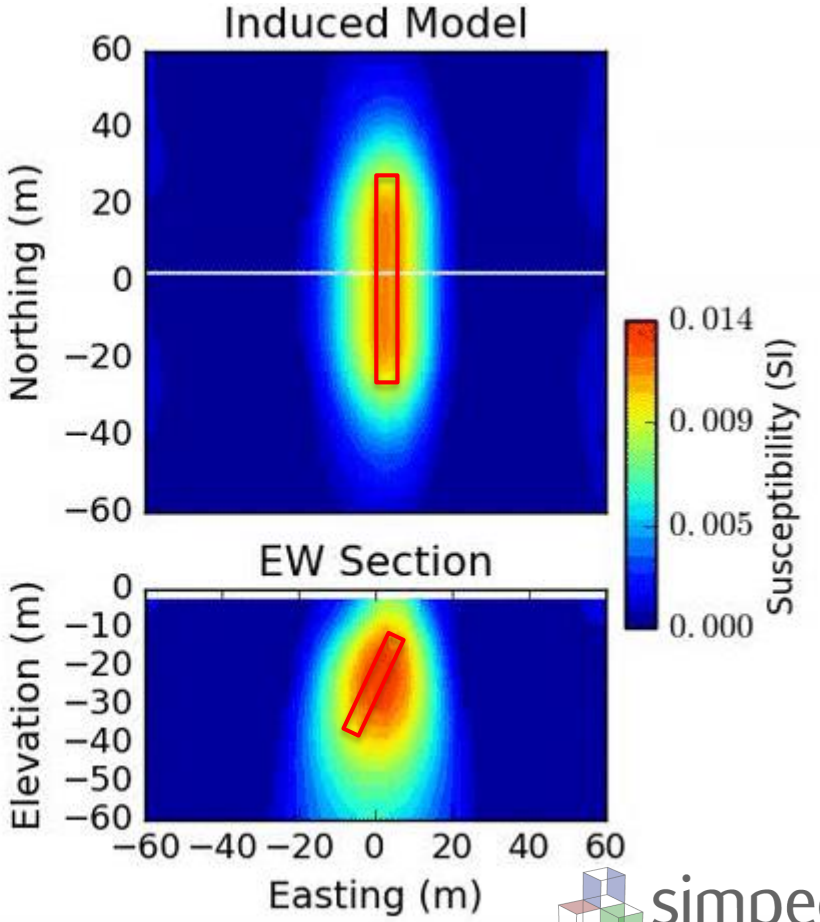
Magnetic Susceptibility Inversion

- Induced assumption: (Li & Oldenburg 1996)



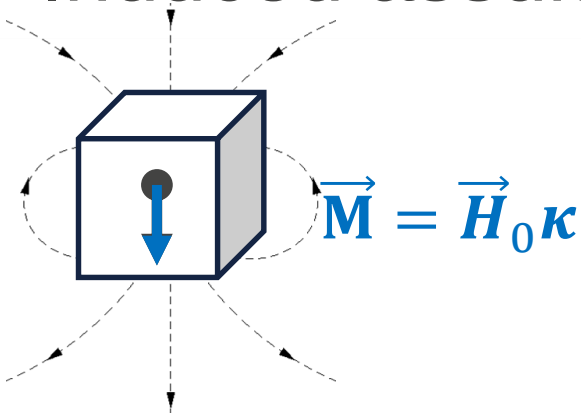
$$\vec{b} = \mathbf{T}\vec{\mathbf{H}}_0 \kappa$$





Magnetic Susceptibility Inversion

- Induced assumption

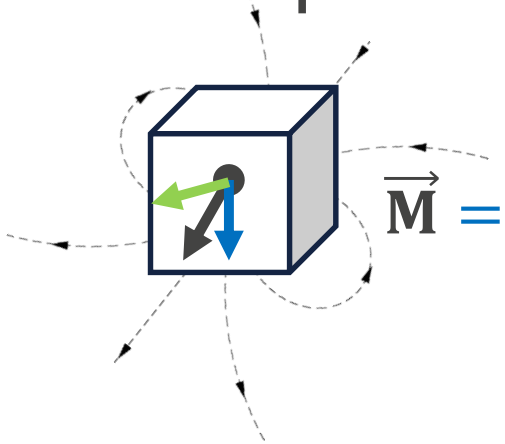


$$\vec{b} = \mathbf{T}\vec{\mathbf{H}}_{\mathbf{0}}\kappa$$

  - Stable and robust for any inducing field orientation
  - Several commercial (MAG3D) and open-source codes available
  - Extensively used by the mineral exploration sector.

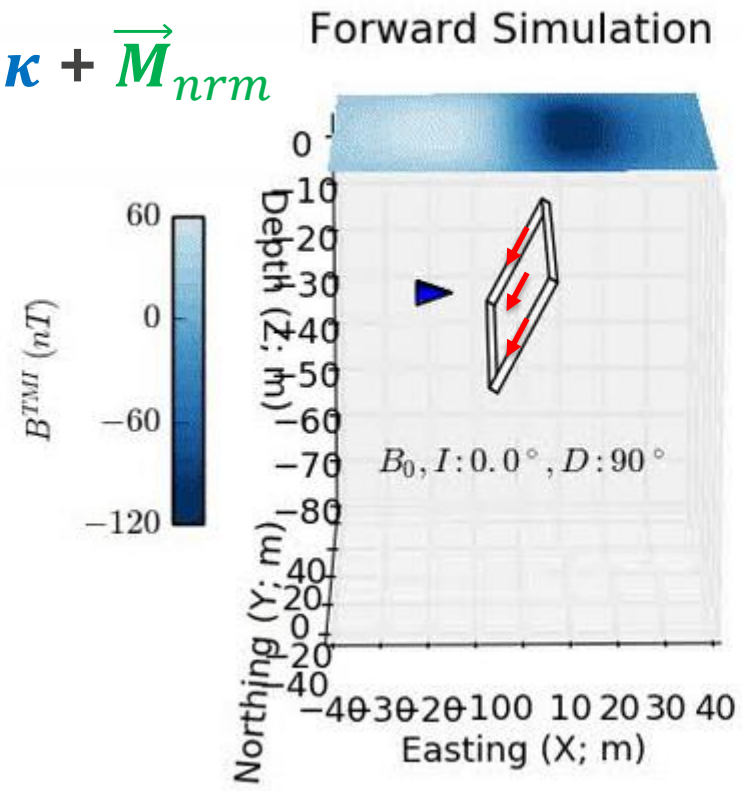
Magnetic Susceptibility Inversion

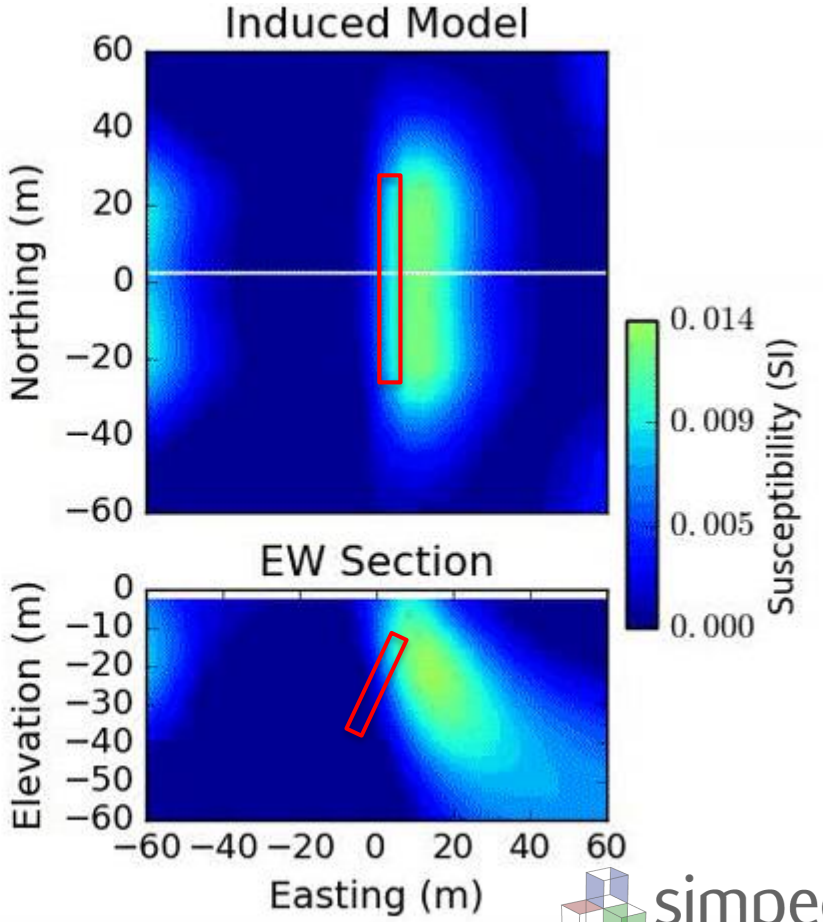
- Complication with remanence



$$\vec{\mathbf{M}} = \vec{\boldsymbol{H}}_0\boldsymbol{\kappa} + \vec{\boldsymbol{M}}_{nrm}$$

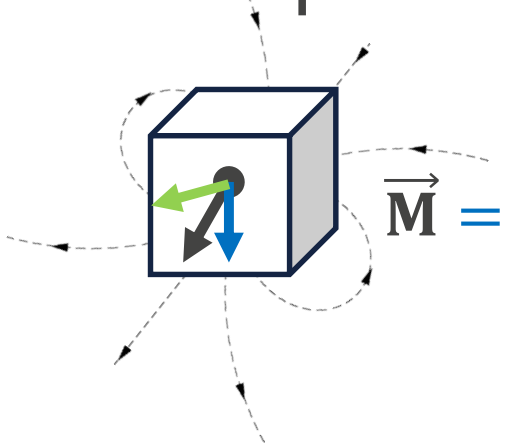
$$\vec{b} = \mathbf{T}\vec{\mathbf{H}}_0\kappa$$





Magnetic Susceptibility Inversion

- Complication with remanence



$$\vec{\mathbf{M}} = \vec{H}_0\kappa + \vec{M}_{nrm}$$

  - Wrong assumption results in severe distortion
  - Common for many types of mineral deposits (Kimberlite, BIF, VMS)

$$\vec{b} = \mathbf{T}\vec{\mathbf{H}}_{\mathbf{0}}\kappa$$

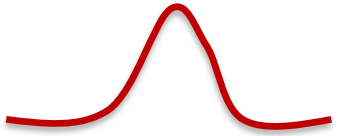
Previous Work

- Data processing
  - Fourier domain (Roest et al. 1992)
  - Helbig's method (Phillips 2003)
  - Cross-correlation (Dannmiller & Li 2006)

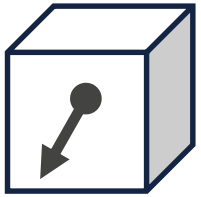
- Simple and fast
- Becomes hard for multiple anomalies and requires gridding

Previous Work

- Amplitude Inversion
  - Invert data weakly sensitive to magnetization



$$|\vec{b}| = \left[{b_x}^2 + {b_y}^2 + {b_z}^2\right]^{1/2}$$



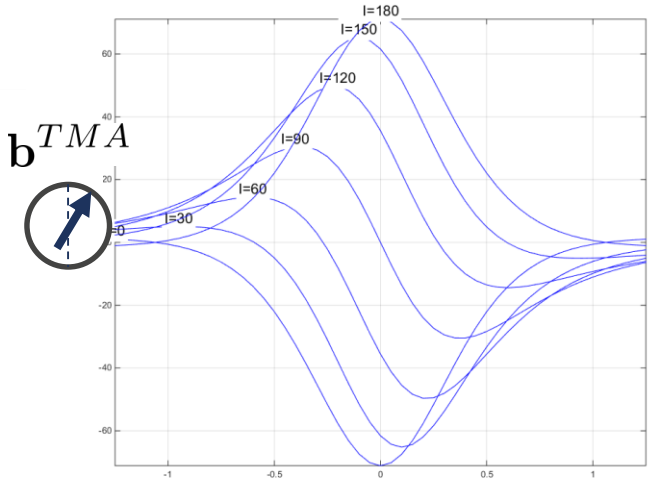
$$\vec{\mathbf{M}} \approx \vec{\boldsymbol{H}}_0 \boldsymbol{\kappa}_e$$

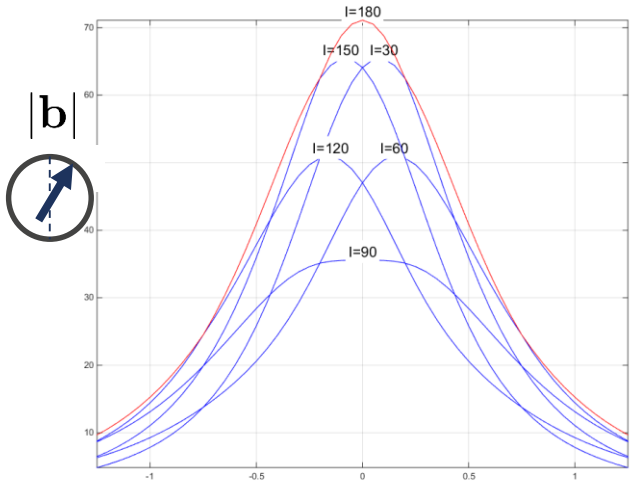
  - **Major work from CSM**

    Sarah Shearer, MSc. Thesis (2005)

    Li et al. 2010

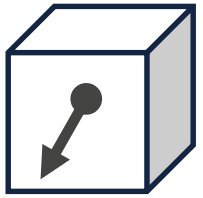
    Li & Li 2014





Previous Work

- Amplitude Inversion: Li et al. 2014

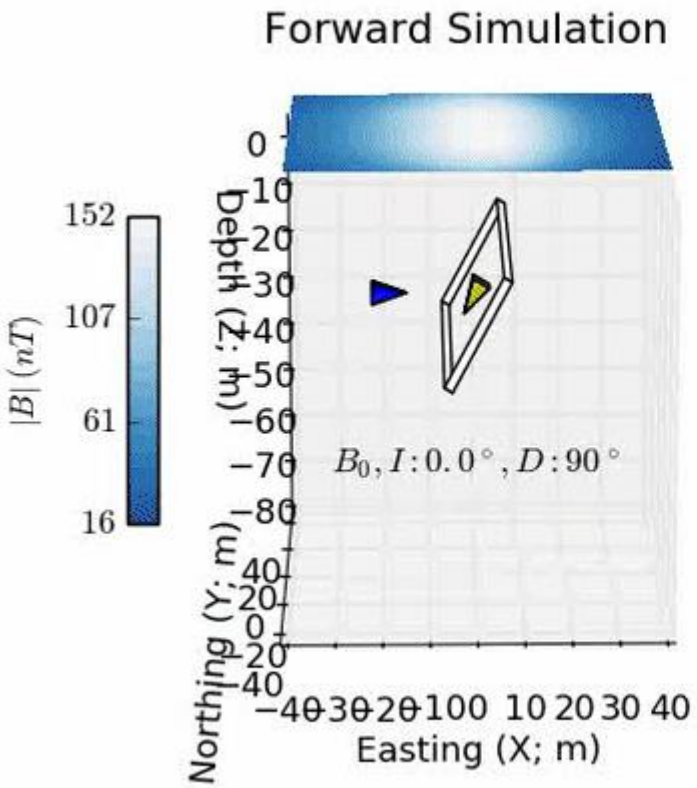


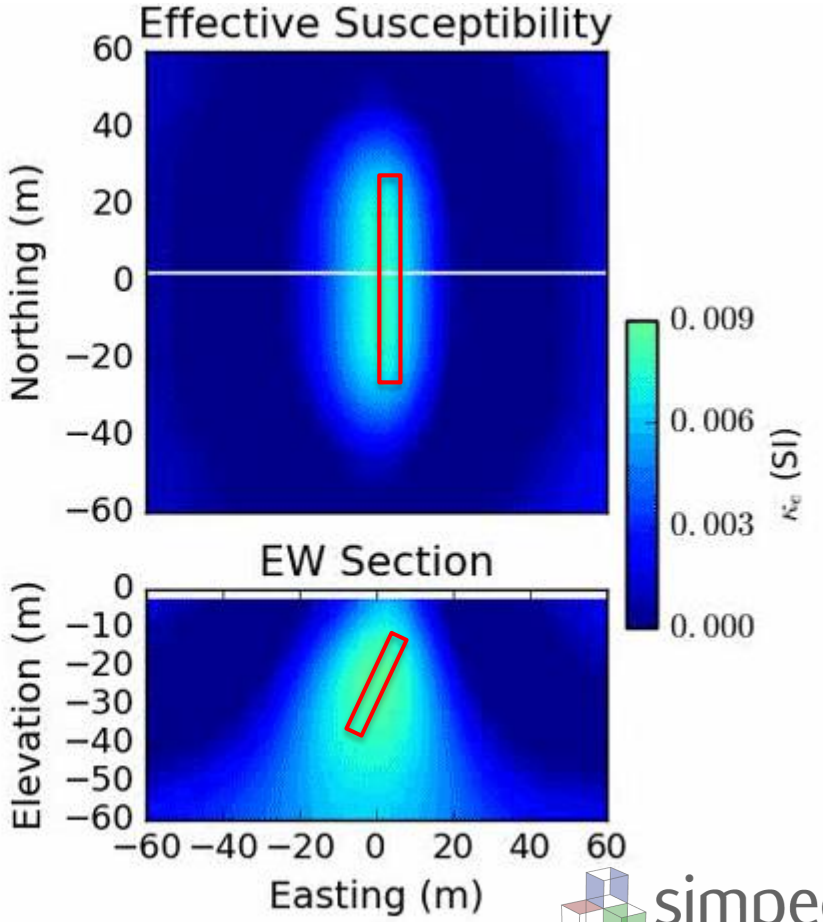
$$\vec{\mathbf{M}} \approx \vec{\boldsymbol{H}}_0 \boldsymbol{\kappa}_e$$

$$|\vec{b}| = \mathbf{F}[m]\kappa_e$$

Sensitivity weights applied

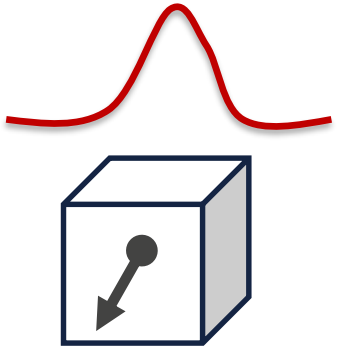
$$W_r = \left[\sum_{i=1}^{nd} J_{[i:]}{}^2\right]^{1/2}$$





Previous Work

- Amplitude Inversion

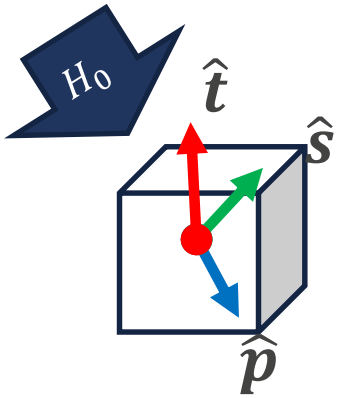


$$|\vec{b}| = \left[{b_x}^2 + {b_y}^2 + {b_z}^2\right]^{1/2}$$

  – Robust recovery of effective susceptibility
  – Requires pre-processing for amplitude data

Previous Work

- Magnetic Vector Inversion, Cartesian (MVI-C)
  - Directly for the components of magnetization



$$\vec{\mathbf{M}}_{pst} = \begin{bmatrix} \mathbf{H}_p & \mathbf{H}_s & \mathbf{H}_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix}$$

  - Lelievre & Oldenburg (2009)
  - Geosoft VOXI

Magnetic Inversion

- Magnetization Vector Inversion, Cartesian (MVI-C)

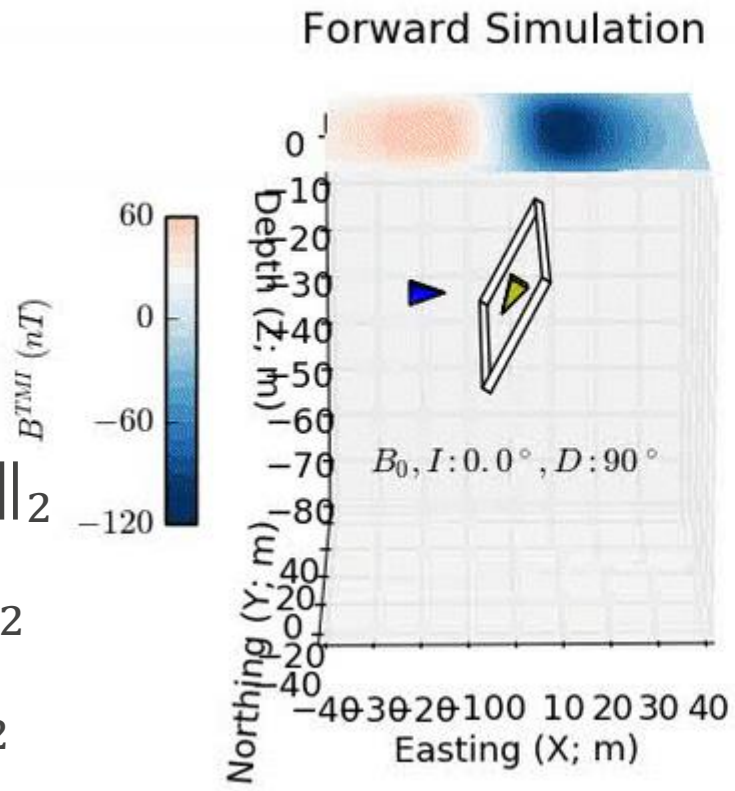
$$\vec{b} = \mathbf{T}\vec{\mathbf{M}}_{pst}$$

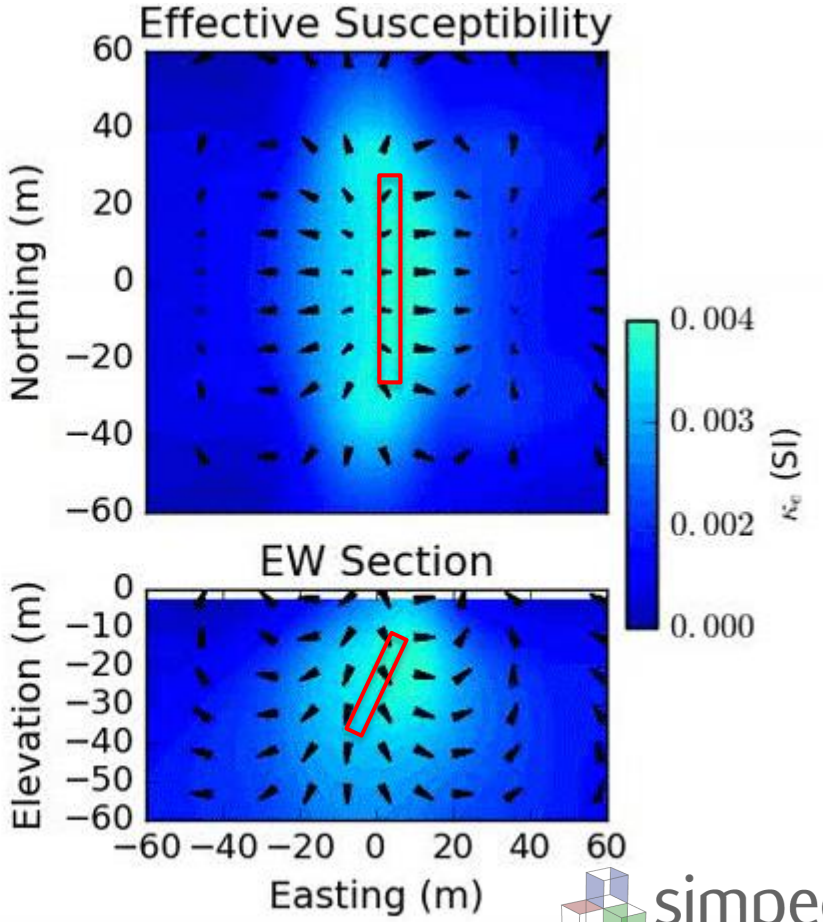
$$\varphi_m = \varphi_p + \varphi_s + \varphi_t$$

$$= \|p - p_{ref}\|_2 + \|\nabla p\|_2$$

$$+ \|s - s_{ref}\|_2 + \|\nabla s\|_2$$

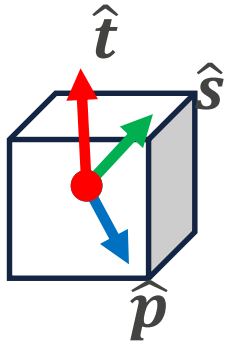
$$+ \|t - t_{ref}\|_2 + \|\nabla t\|_2$$





Previous Work

- Magnetic Vector Inversion, Cartesian (MVI-C)

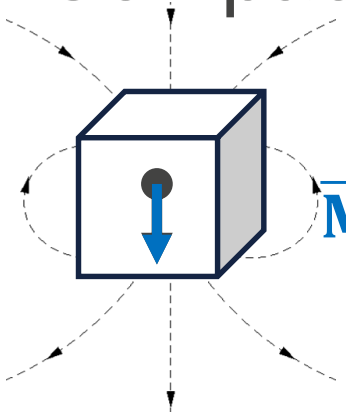


$$\vec{\mathbf{M}}_{pst} = \begin{bmatrix} \mathbf{H}_p & \mathbf{H}_s & \mathbf{H}_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix}$$

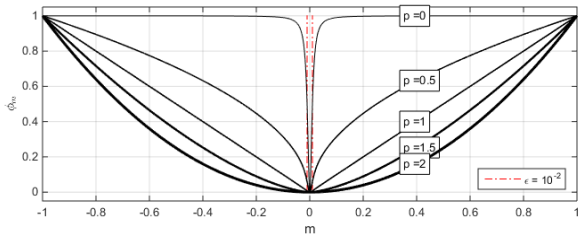
  – Recovers both amplitude and orientation
  – Largely underdetermined, relies heavily on regularization
  – Models too smooth for easy interpretation
  – Clustering methods (FMC) (Li & Sun 2016)

Parallel research

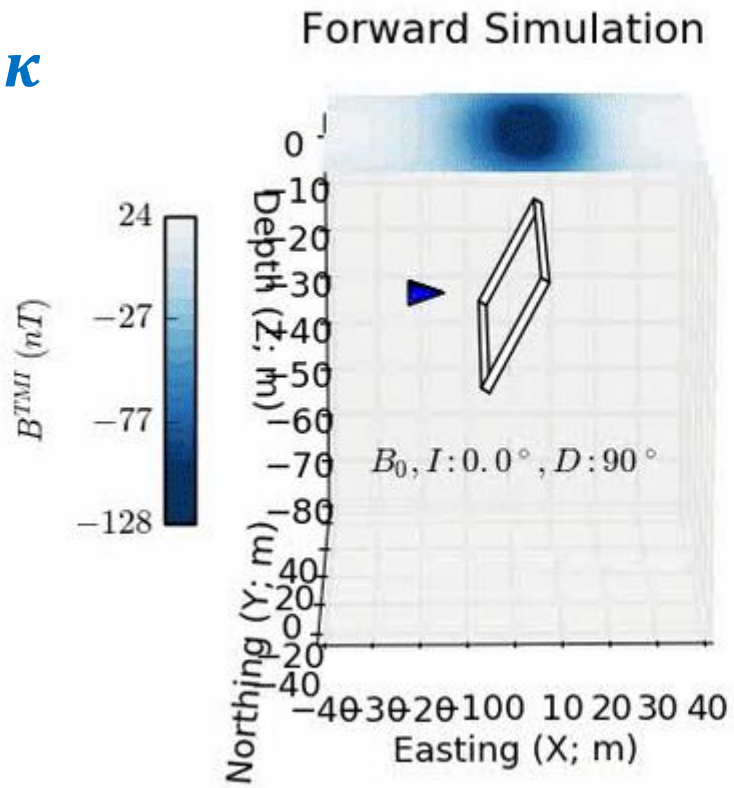
- Compact Norms (Next talk @ 4:10 pm)

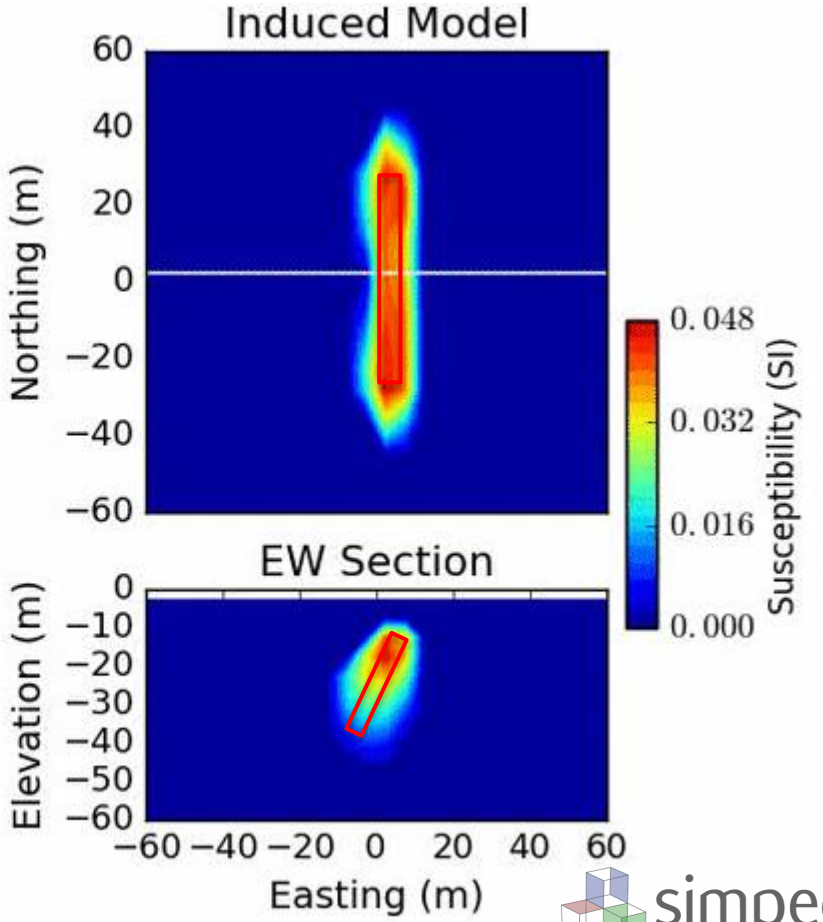


$$\vec{\mathbf{M}} = \vec{H}_0 \boldsymbol{\kappa}$$



$$\|\mathbf{f}(m)\|_p^p = \sum_{i=1}^{nc} |\mathbf{f}(m_i)|^p$$

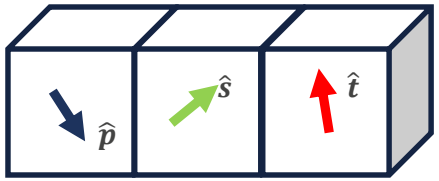




Parallel research

- Limitations with current formulation
  - Sparsity imposed on individual components favoring magnetization along single component:

$$\vec{\mathbf{M}}_{pst} = \begin{bmatrix} \mathbf{H}_p & \mathbf{H}_s & \mathbf{H}_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix}$$



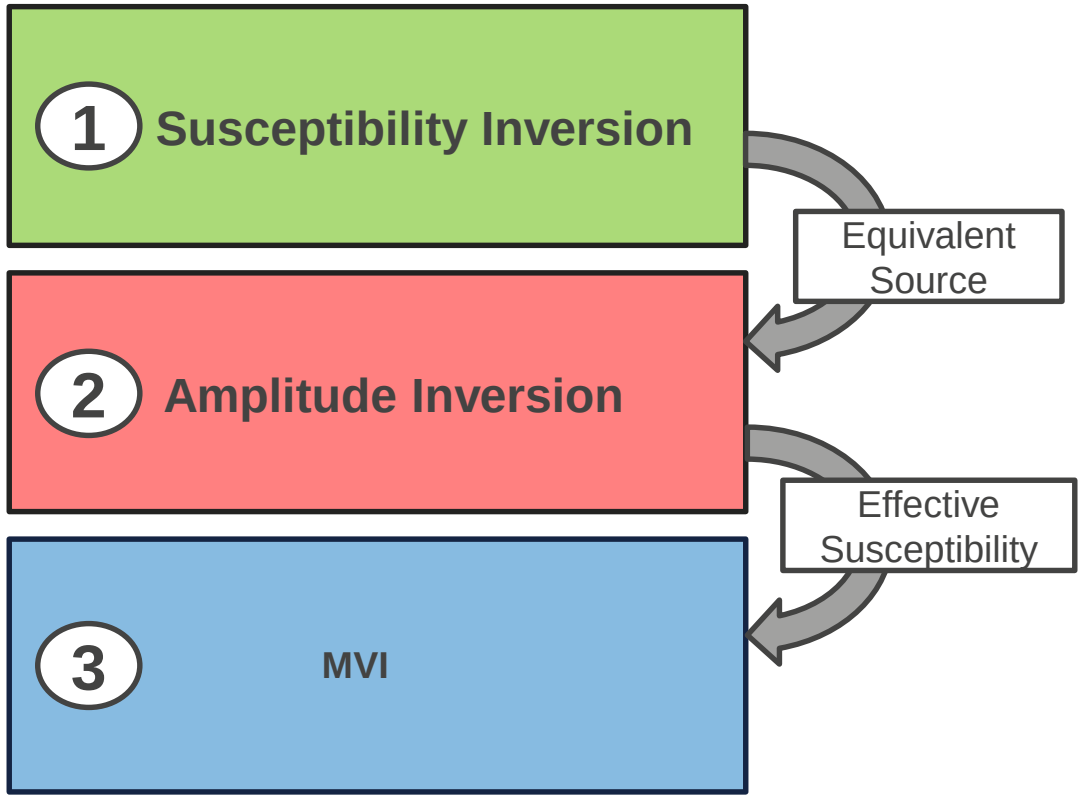
  - Difficult to incorporate borehole constraints (koenigsberger ratio)

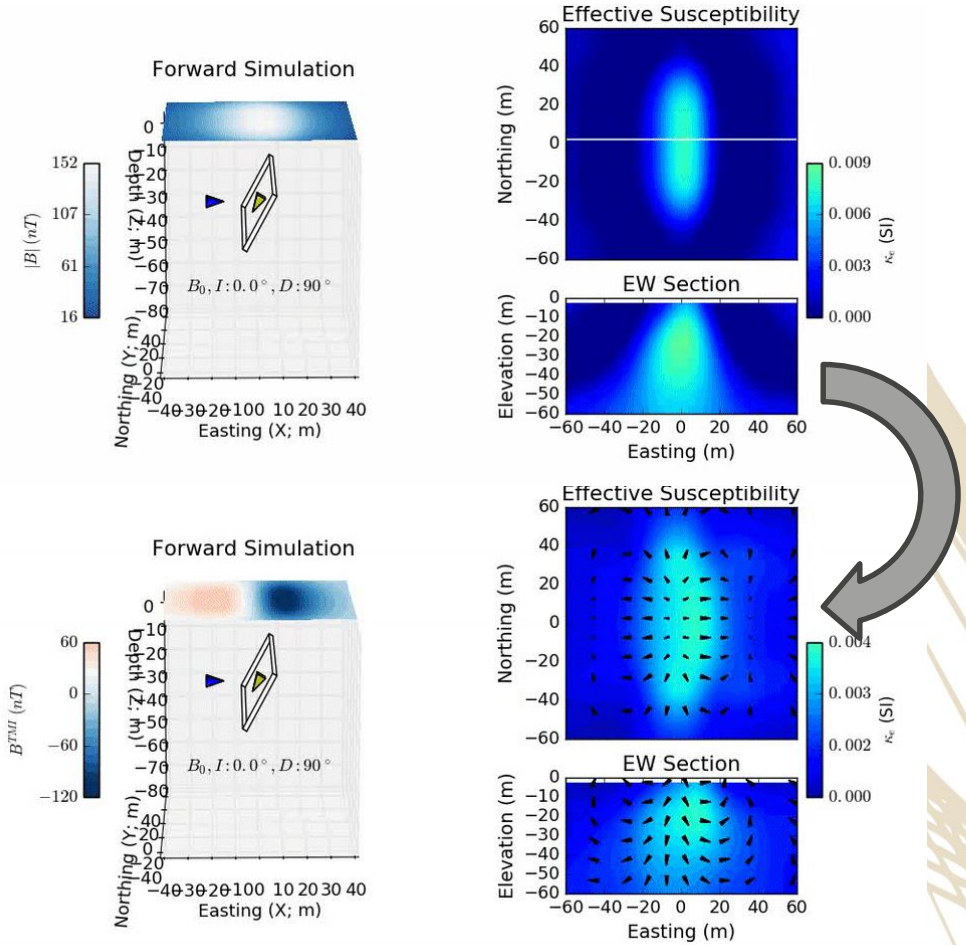
Cooperative Strategy

- Cooperative Magnetic Inversion (CMI)

Fournier, MSc. 2015

Liu et al. 2015





Cooperative Strategy

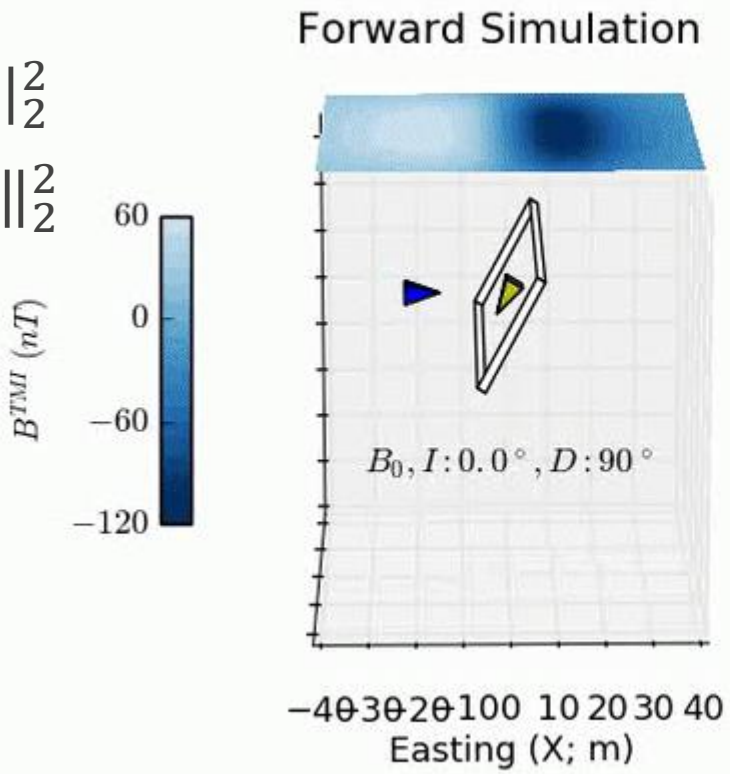
- Cooperative Magnetic Inversion (CMI)

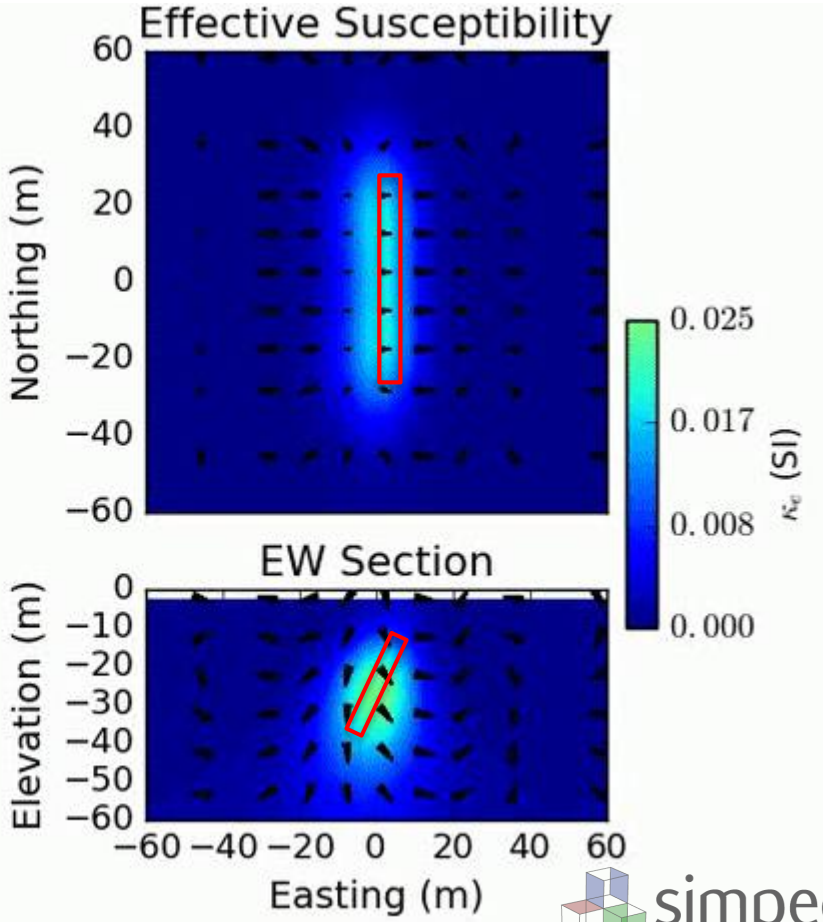
Weighted regularization

$$\varphi_m = \|\mathbf{W}_m \mathbf{W}_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \sum_{i=x,y,z} \|\mathbf{W}_m \mathbf{W}_i \mathbf{G}_i \mathbf{m}\|_2^2$$

Model weight

$$W_m = \left[\frac{\kappa_e}{\max(\kappa_e)} + \varepsilon\right]^{-1}$$

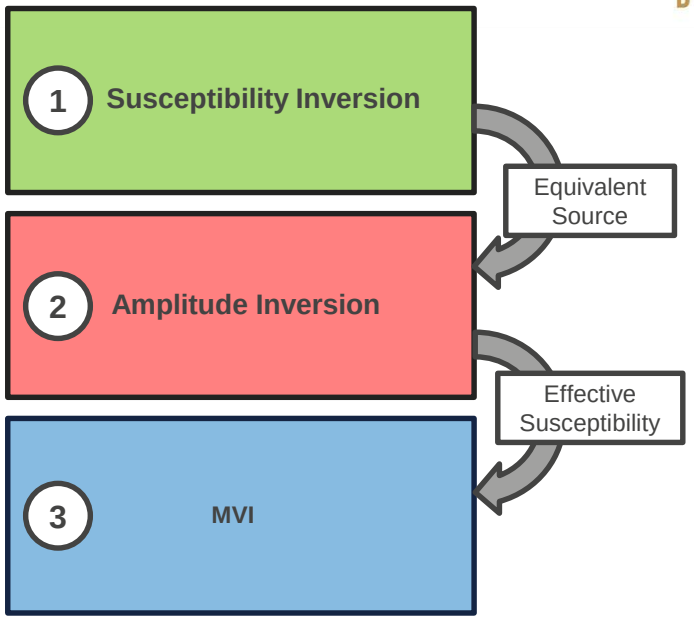




Cooperative Strategy

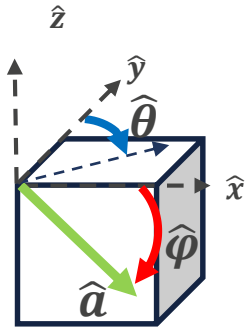
- Cooperative Magnetic Inversion (CMI)
  - Able to apply sparsity
  - Recover both the amplitude and direction

  - Minimum of 3 inversions required
  - Dependents on the ability of extracting amplitude data
  - Can’t apply sparsity on the orientation



Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Separate the amplitude and direction

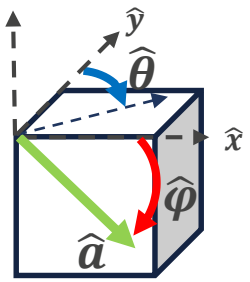


$$\hat{x} = a\cos\varphi\cos\theta$$
$$\hat{y} = a\cos\varphi\sin\theta$$
$$\hat{z} = a\sin\varphi$$

  - Lelievre's PhD. (2009)
  - Put aside due to high non-linearity

Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Natural separate the amplitude and direction



$$\hat{x} = a\cos\varphi\cos\theta$$
$$\hat{y} = a\cos\varphi\sin\theta$$
$$\hat{z} = a\sin\varphi$$

  - Solved issues regarding the non-linearity

$$\varphi_m = \varphi_a + \varphi_t + \varphi_p = \|a - a_{ref}\|_2 + \|\nabla a\|_2 + \|t - t_{ref}\|_2 + \|\nabla t\|_2 + \|p - p_{ref}\|_2 + \|\nabla p\|_2$$

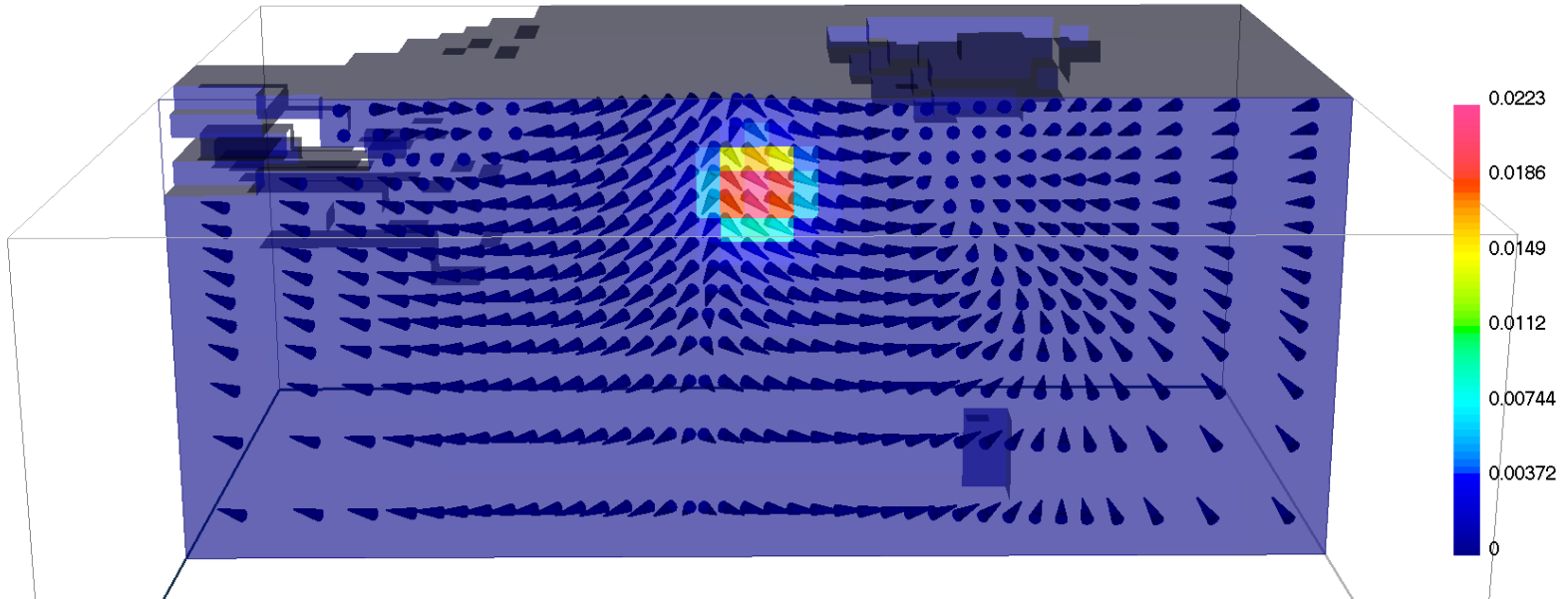
Sensitivity: $J = \frac{\partial d}{\partial m} = SG$

$$S = \begin{bmatrix} \cos\varphi\cos\theta & -a\sin\varphi\cos\theta & -a\cos\varphi\sin\theta \\ \cos\varphi\sin\theta & -a\sin\varphi\sin\theta & a\cos\varphi\cos\theta \\ \sin\varphi & \cos\varphi & 0 \end{bmatrix}$$

Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Apply Koenigsberger ratio constraints

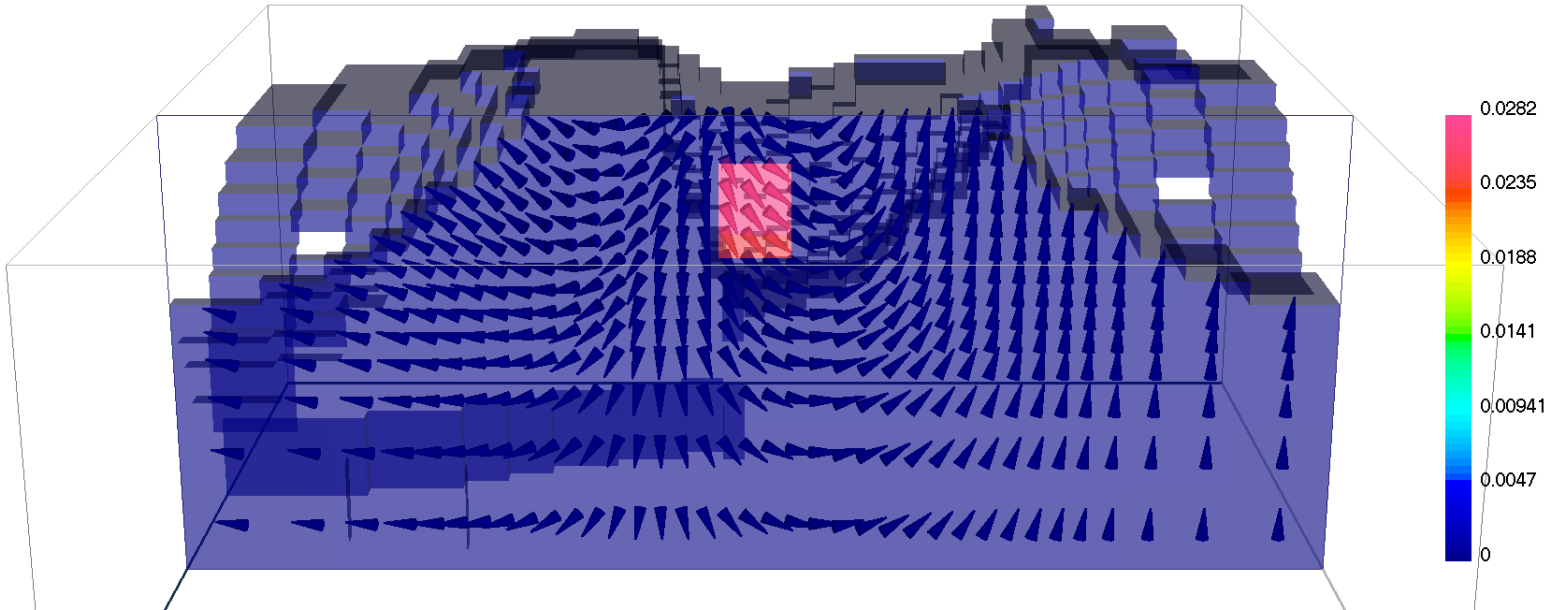
$$\varphi_m = \|a - \boxed{a_{ref}}\|_2 + \|\nabla a\|_2 + \|\nabla t\|_2 + \|\nabla p\|_2$$



Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Sparsity on amplitude

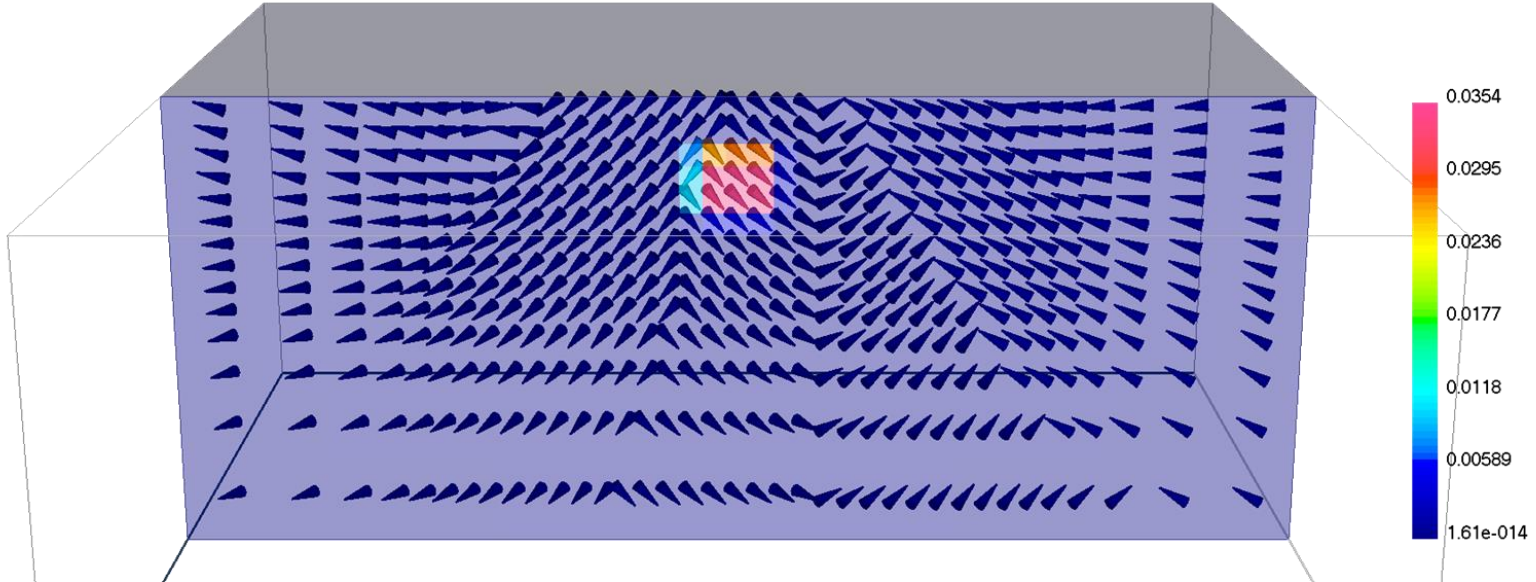
$$\varphi_m = \boxed{\|\mathrm{a} - \mathrm{a}_{\mathrm{ref}}\|_{\mathrm{p}} + \|\nabla \mathrm{a}\|_{\mathrm{p}}} + \|\nabla \mathrm{t}\|_2 + \|\nabla \mathrm{p}\|_2$$



Ongoing research

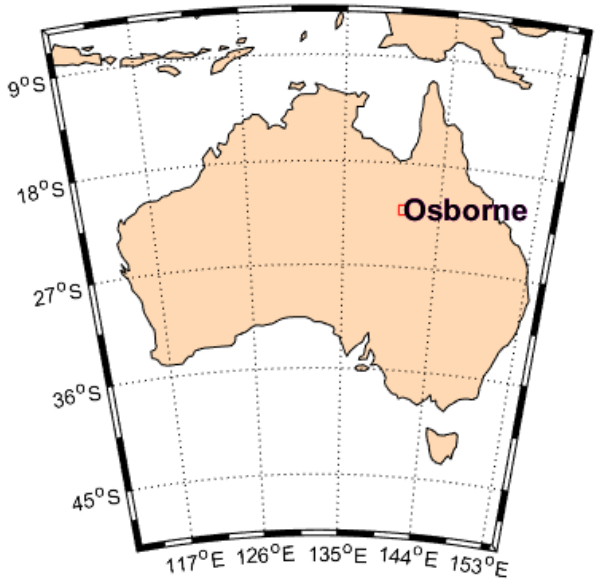
- Magnetic Vector Inversion, Spherical (MVI-S)
  - Sparsity on angles

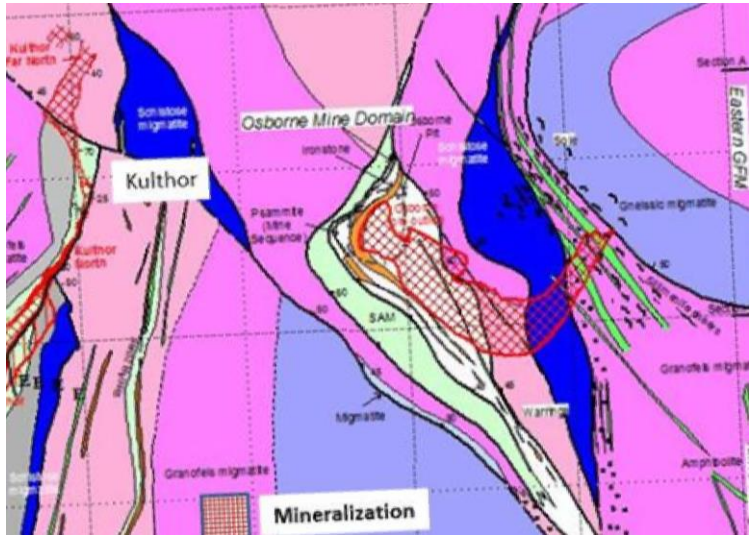
$$\varphi_m = \|\mathrm{a} - \mathrm{a}_{\mathrm{ref}}\|_{\mathrm{p}} + \|\nabla \mathrm{a}\|_{\mathrm{p}} + \boxed{\|\nabla \mathrm{t}\|_{\mathrm{p}} + \|\nabla \mathrm{p}\|_{\mathrm{p}}}$$



Case History

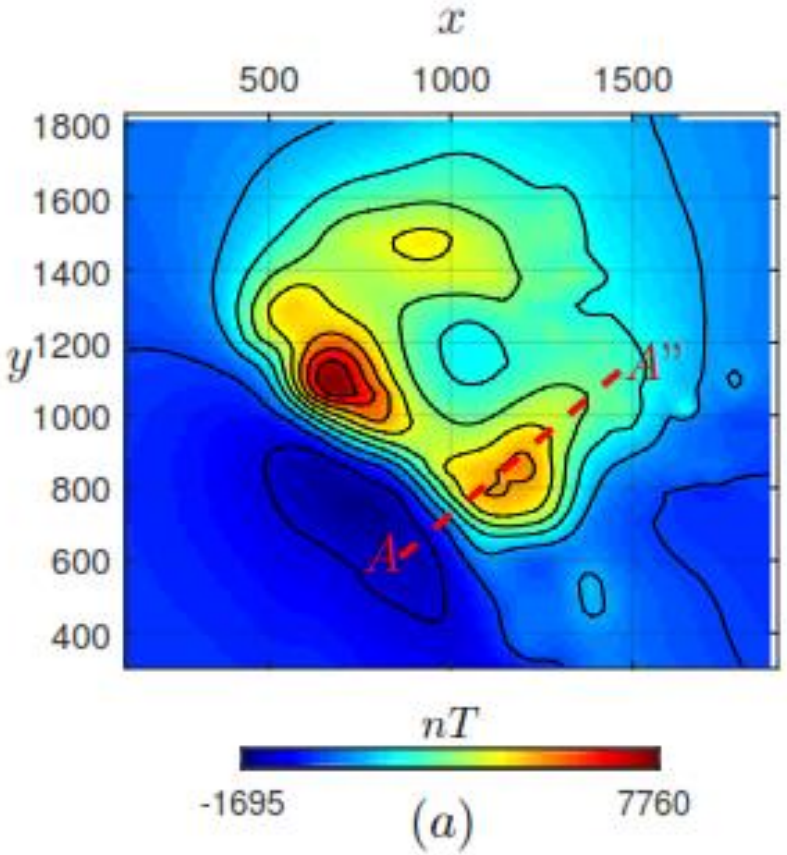
- Osborne Cu-Au deposit, Queensland
  - Airborne mag discovery in 1974 (Newmont Pty Ltd)
  - Magnetite-rich Ironstones layers within metasediments of Mt Isa quartzite.

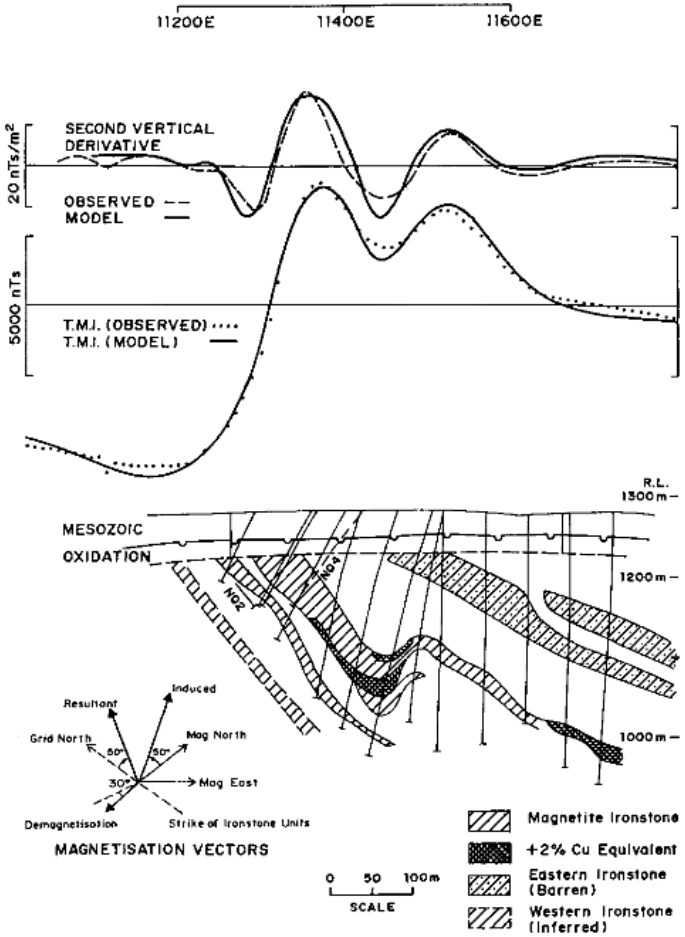




Case History

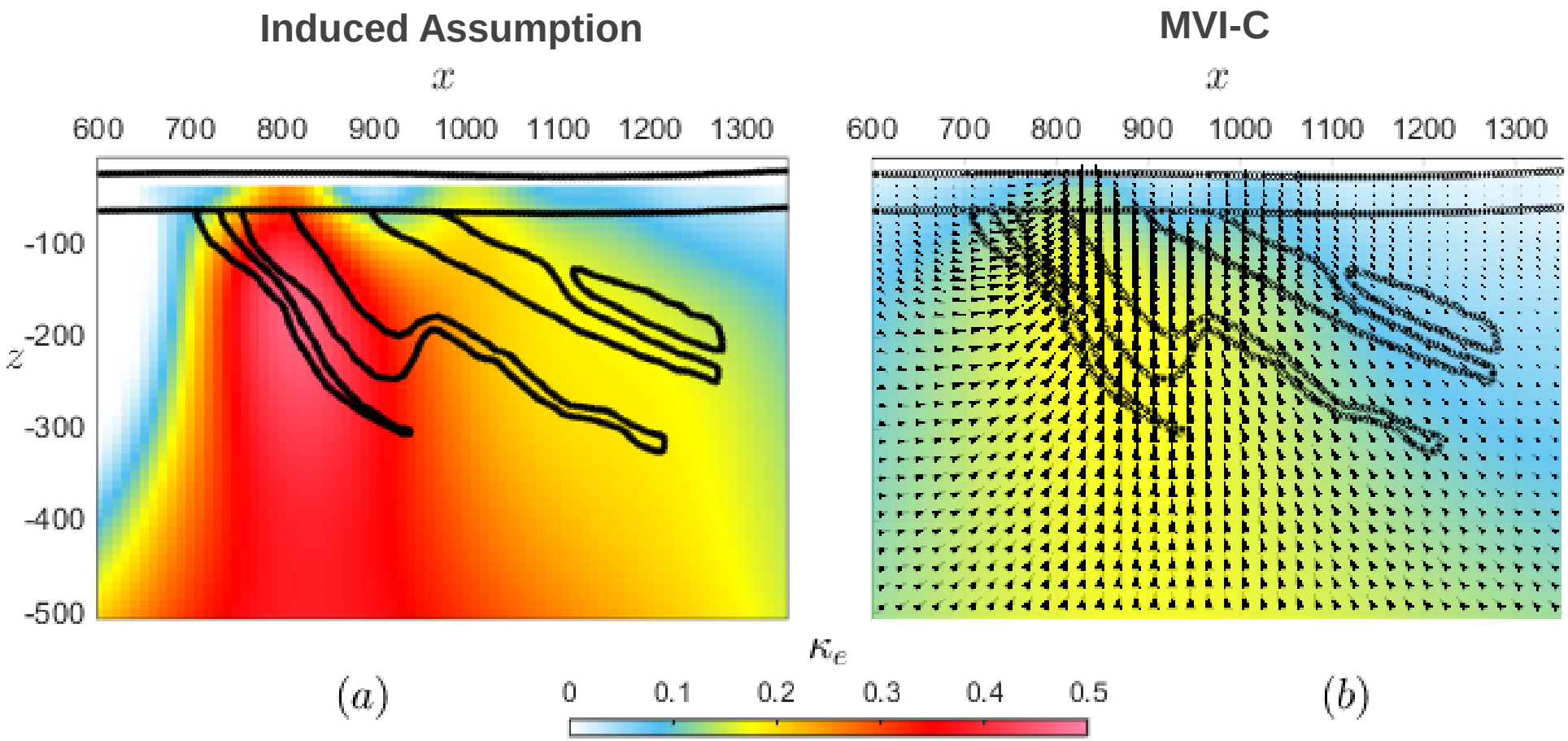
- Strong self-demagnetization effects as studied by Logan & Anderson (1992)





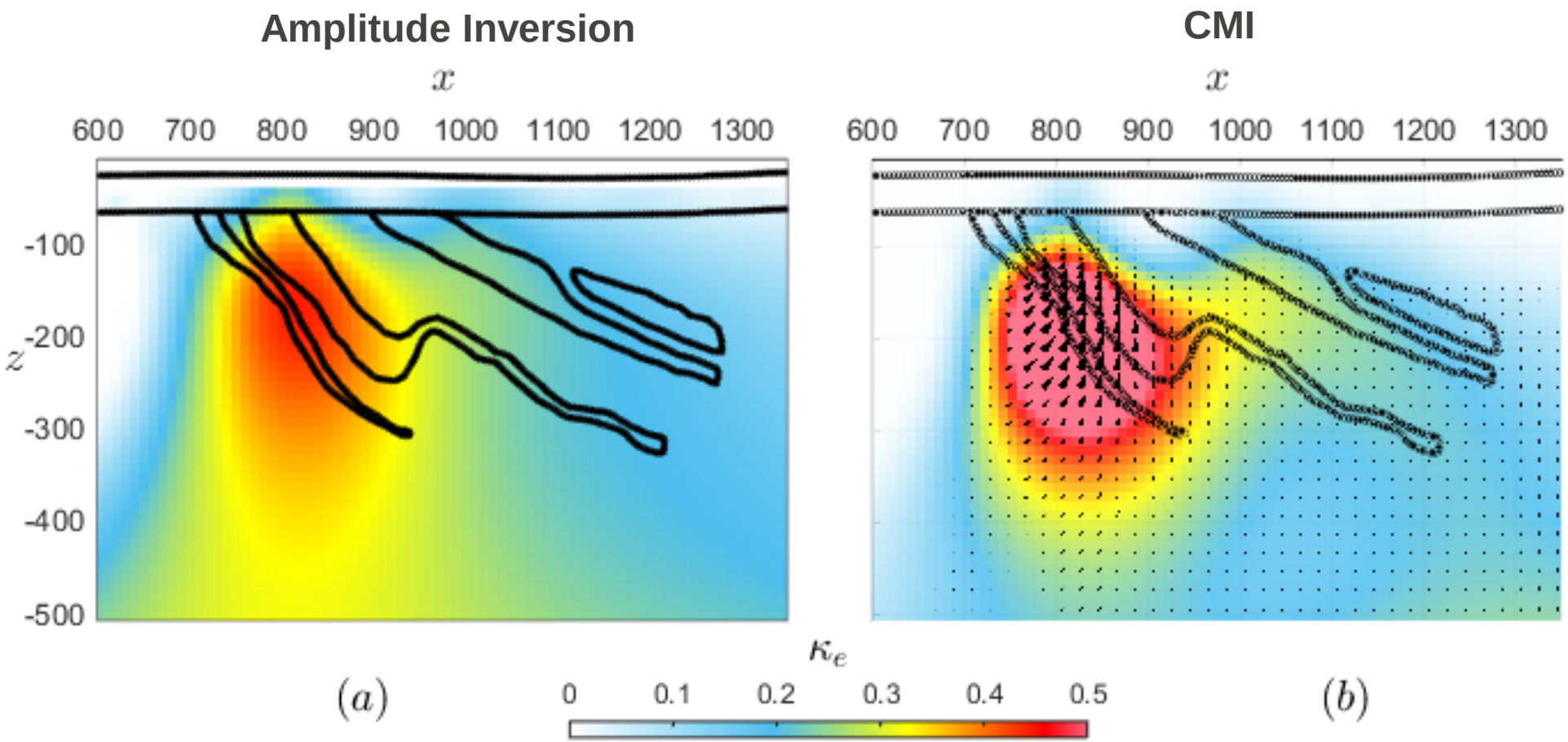
Case History

- Conventional Inversions



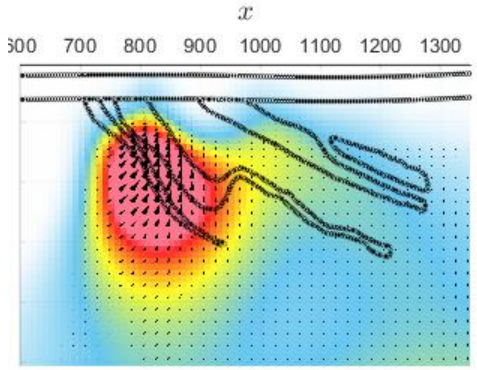
Case History

- Conventional Inversions

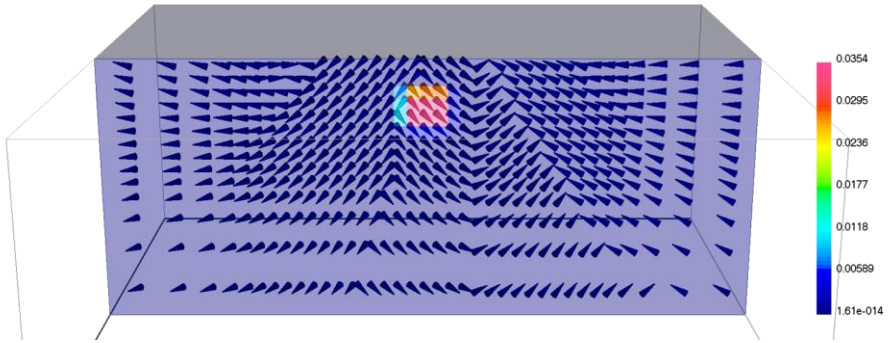


Summary

- Managed to reduce the non-uniqueness of usual MVI-Cartesian formulation
- Allow to apply sparse norms



- Promising results with MVI-Spherical formulation
- Requires more work to improve performance



Thank you!