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# Magnetic and Electric Fields of Direct Currents in a Layered Earth (Short Note)

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**Key Words**: magnetometric resistivity, MMR, 1D, direct current, magnetic field, electric field

## ABSTRACT

We derive a numerical solution of the steady-state magnetic field due to a DC current source in a layered earth model. Such a solution is critical for interpretation of magnetometric resistivity (MMR) data. Our solution is achieved by solving a boundary value problem in the spatial wavenumber domain and then transforming back to the polar spatial domain. The propagator matrix technique is used to interrelate solutions between layers. Two simplified cases are examined to promote understanding of the magnetic fields. In particular, the derived formula illustrates why surface MMR data are insensitive to 1D conductivity variations and why borehole measurements are effective in finding conductivity contrasts.

## INTRODUCTION

The magnetometric resistivity method (MMR) has recently become an additional electrical prospecting technique used for finding mineral resources (e.g., Bishop et al., 1997). This technique is based on the measurement of low-frequency magnetic fields associated with non-inductive current flow in the ground (which is numerically regarded as "direct current" or DC). In contrast to the commonly used DC resistivity method in which electric fields are measured, MMR has its own unique characteristics. For example, surface MMR data are insensitive to 1D variation of conductivity, but down-hole MMR data are dependent upon layered structure. For interpreting downhole data, we are required to separate the observed data into a "background" field (we call it $B^{1d}$ in this note), which is attributed to the 1D background host material, and an anomalous response due to a 2D or 3D target body. Our experience has shown that a good understanding of the background field in a layered earth is critical to the success of a 3D inversion of field data (Chen et al., 2004).

As pointed out by Veitch et al. (1990), the general solution for the magnetic field within a layered earth excited by a point source has not been fully explored, although some researchers have addressed this problem. For example, Edwards et al. (1985) discuss a specific case where the upper half-space is conductive seawater, as encountered in the magnetometric offshore electrical sounding (MOSES) system; Edwards (1988) and Edwards and Nabighian (1991) concentrate upon estimating the ratio of the magnetic fields below and above a known conductive layer to infer the basement resistivity; Sezginer and Habashy (1988) use an image method for computing the static magnetic field due to an arbitrary current injected into a conducting uniform half-space; Inayat-Hussein (1989) gives a new proof that the magnetic field outside the 1D medium is independent of the electrical conductivity; Veitch et al. (1990) "indirectly" derive the magnetic field by applying Stokes' theorem and Ampère's law to the electric potential, which was presented by Daniels (1977). Unfortunately, these works are not sufficiently general about the magnetic field to be used for many current applications. We elaborate upon these next.

Theoretically, the 3D conductivity of the earth can be recovered (to within a multiplicative constant) by inverting MMR data. In mineral exploration problems, however, data are acquired by having only a few current locations, and measurements of the magnetic field are made on the surface and in a few boreholes. In such circumstances, direct application of a 3D inversion algorithm does not produce a highly resolved conductivity distribution, and further restriction of model space and incorporation of prior information is required. In particular, the inversion produces much better results when looking for 3D targets within a known 1D background. The critical importance of background conductivity on the recovery of 3D targets requires that we develop a 1D MMR forward modelling and inversion solution (Chen et al., 2004). In practice, it is possible to use the 1D frequency-domain solution for a grounded wire source with the frequency set "sufficiently" low. The frequency domain solution has been extensively studied (Wait, 1982; Ward and Hohmann, 1991; Sorenson and Christensen, 1994). However, it is more satisfactory to have a direct solution rather than relying on asymptotic behaviour. A second reason for pursuing our solution is that it forms the basics for development of a formula by which to estimate apparent resistivity in marine MMR (Chen and Oldenburg, 2004), which allows first order information about conductivity of the seafloor to be obtained directly from the data. The apparent resistivity is defined from the azimuthal component of magnetic field at the sea bottom. This magnetic field is related to the relative geometric positions of transmitter wire and the observation point, the thickness of seawater, and the resistivities of the two-layered model (including the seawater). This requires an explicit expression between the magnetic field and the seafloor resistivity, and the ability to take into account the geometric difference between the transmitter wire and receiver that arises because of the bathymetry of the sea floor.

In this note, we derive the magnetic field directly from solving a boundary value problem, similar to the approach used by Edwards (1988), and then briefly discuss simplifications in a homogeneous and two-layered earth. The solution is compared with a published example.

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## MAGNETIC FIELD DUE TO A SEMI-INFINITE SOURCE IN A 1D EARTH

As shown in Figure 1, a semi-infinite vertical wire *AOC* carries an exciting current $I$ and terminates at the location $C$. The electrode $C$ is placed deliberately at the interface $z = z_s$ of layer $s$ and layer $s$+1 to simplify the mathematics. Each layer has a constant conductivity $\sigma_j$ with thickness $h_j$ for the upper $N$–1 layers. In a source-free region, the magnetic field **H** can be written as

$$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{H} = \mathbf{0} \quad . \tag{1}$$

The problem is axisymmetric, and **H** has only an azimuthal component in cylindrical coordinates $(r, \phi, z)$. For simplicity, we use $H$ to represent the azimuthal component in the following derivations. Expanding equation (1) yields

$$\frac{\partial^2 H}{\partial^2 r} + \frac{1}{r}\frac{\partial H}{\partial r} - \frac{1}{r^2} H + \sigma \frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial H}{\partial z} + \frac{\partial^2 H}{\partial^2 z} = 0 \quad . \tag{2}$$

Since $\sigma$ is a constant in each layer, the fourth term in equation (2) can be deleted so

$$\frac{\partial^2 H}{\partial^2 r} + \frac{1}{r}\frac{\partial H}{\partial r} - \frac{1}{r^2} H + \frac{\partial^2 H}{\partial^2 z} = 0 \quad . \tag{3}$$

This expression is the same as equation (6) in Edwards and Nabighian (1991), except that we use field $H$ instead of $B$. The advantage of using $H$ is that it is independent of the magnetic permeability in this unique situation. If values of $B$ are required, they can be obtained by computing $B = \mu H$, where $\mu$ is the magnetic permeability at the observation location. In fact $\mu$ can be any function of $z$. Because of the symmetry, $H$ has only an azimuthal component, and there is no magnetization or "magnetic charges" accumulated at the interfaces of different layers. Therefore, we only need to solve for magnetic field from equation (1). The conservative law $\nabla \cdot \mathbf{B} = \mathbf{0}$ is automatically satisfied for our problem.

Following Edwards and Nabighian (1991), we define a Hankel transform pair as

$$\tilde{H}(\lambda, z) = \int_0^\infty rH(r,z) J_1(\lambda r) dr \tag{4}$$

and

$$H(r,z) = \int_0^\infty \lambda \tilde{H}(\lambda, z) J_1(\lambda r) d\lambda \quad , \tag{5}$$

where $J_1$ is the Bessel function of the first kind of order one. The Hankel transform of equation (3) results in the simple second order equation

$$\frac{\partial^2 \tilde{H}}{\partial^2 z} - \lambda^2 \tilde{H} = 0 \quad , \tag{6}$$

where $\tilde{H}$ is the magnetic field in wavenumber $\lambda$ domain. A complementary solution to equation (6) in any layer $j$ is

$$\tilde{H}_j(\lambda, z) = \tilde{D}_j e^{-\lambda(z - z_{j-1})} + \tilde{U}_j e^{\lambda(z - z_{j-1})} \quad , \tag{7}$$

where $\tilde{D}_j$ and $\tilde{U}_j$ are the downward and upward propagation coefficients, independent of the variable $z$, which can be determined through a propagator matrix by applying the boundary conditions. In order to determine $\tilde{D}_j$ and $\tilde{U}_j$, we use the boundary conditions: the azimuthal component of $H$ field and the radial component of $E$ are continuous across the interface, i.e.,

$$\tilde{H}_j(\lambda, z)\big|_{z=z_j} = \tilde{H}_{j+1}(\lambda, z)\big|_{z=z_j} \tag{8}$$

and

$$\tilde{E}^r_j(\lambda, z)\big|_{z=z_j} = \tilde{E}^r_{j+1}(\lambda, z)\big|_{z=z_j} \quad . \tag{9}$$

The radial and vertical components of $E$ are related to $H$ through

$$E_r = -\frac{1}{\sigma}\frac{\partial H}{\partial z} \tag{10}$$

and

$$E_z = \frac{1}{\sigma r}\frac{\partial}{\partial r}(rH) \quad . \tag{11}$$

In addition, we also have to use two other constraints, i.e.,

$$\tilde{U}_N = 0 \quad , \tag{12}$$

and no current crosses the air-earth interface

$$\sigma E_z\big|_{z=0} = 0 \quad . \tag{13}$$

Therefore, the $2N$ unknown coefficients can be determined from the $2(N$–$1)$+2 equations.

### A General Solution for a Two-layered Earth

It is insightful to derive the solutions for a 1D earth that has a top layer of conductivity $\sigma_1$ overlying a half-space of conductivity $\sigma_2$. The current electrode is located at the interface $z = z_1$. In layer 1, $\tilde{H}_1$ consists of two parts (Wait, 1982); one is the complementary solution $\tilde{H}^s_1$, shown in equation (7), and the other part is the particular solution $\tilde{H}^p_1$, which is

$$\tilde{H}^p_1(\lambda, z) = \frac{I}{2\pi\lambda} \quad . \tag{14}$$

Therefore, $\tilde{H}_1$ is given by

$$\tilde{H}_1(\lambda, z) = \frac{I}{2\pi\lambda} + \tilde{D}_1 e^{-\lambda z} + \tilde{U}_1 e^{\lambda z} \quad . \tag{15}$$

In the second layer,

$$\tilde{H}_2(\lambda, z) = \tilde{D}_2 e^{-\lambda(z - z_1)} + \tilde{U}_2 e^{\lambda(z - z_1)} \quad . \tag{16}$$

Use of the above boundary conditions allows us to obtain the four coefficients

$$\tilde{D}_1 = \frac{I}{2\pi\lambda} \frac{e^{-\lambda h_1}}{\left(1 - e^{-2\lambda h_1}\right) + \frac{\sigma_2}{\sigma_1}\left(1 + e^{-2\lambda h_1}\right)} \tag{17}$$

$$\tilde{U}_1 = -\tilde{D}_1 \tag{18}$$

$$\tilde{D}_2 = \frac{I}{2\pi\lambda} \frac{\sigma_2}{\sigma_1} \frac{\left(1 + e^{-2\lambda h_1}\right)}{\left(1 - e^{-2\lambda h_1}\right) + \frac{\sigma_2}{\sigma_1}\left(1 + e^{-2\lambda h_1}\right)} \tag{19}$$

$$\tilde{U}_2 = 0 \quad . \tag{20}$$

A special case is when $\sigma_1 = \sigma_2$, and $h_1$ is the buried depth of the electrode. In this uniform half-space case,

$$\tilde{H}(\lambda, z) = \frac{I}{4\pi\lambda}\left[2 + e^{-2\lambda(z + h_1)} - e^{-2\lambda(h_1 - z)}\right] \quad . \tag{21}$$

Making use of the integral

$$\int_0^\infty e^{-\lambda z} J_1(\lambda r)\,d\lambda = \frac{1}{r}\left(1-\frac{z}{\sqrt{r^2+z^2}}\right) \tag{22}$$

we can transform equation (21) back to spatial domain and obtain

$$H(r,z) = \frac{I}{4\pi r}\left[2-\frac{z+h_1}{\sqrt{r^2+\left(z+h_1\right)^2}}-\frac{z-h_1}{\sqrt{r^2+\left(z-h_1\right)^2}}\right]. \tag{23}$$

**N-layered Earth with a Source in Layer s** ($1 \le s \le N-1$)

If the observation point is located in the source-free region ($s+1 \le j \le N-1$), the coefficients $\widetilde{D}_j$ and $\widetilde{U}_j$ in layer $j$ can be determined from $\widetilde{D}_{j+1}$ and $\widetilde{U}_{j+1}$ in the lower, $j+1$, layer through the relationship

$$\begin{pmatrix}\widetilde{D}_j\\ \widetilde{U}_j\end{pmatrix} = e^{\lambda h_j}\mathbf{\Gamma}_{j+1}\begin{pmatrix}\widetilde{D}_{j+1}\\ \widetilde{U}_{j+1}\end{pmatrix}, \tag{24}$$

where the propagator matrix is

$$\mathbf{\Gamma}_{j+1} = \begin{pmatrix}\frac{1}{2}\left(1+\frac{\sigma_j}{\sigma_{j+1}}\right) & \frac{1}{2}\left(1-\frac{\sigma_j}{\sigma_{j+1}}\right)\\ \frac{1}{2}\left(1-\frac{\sigma_j}{\sigma_{j+1}}\right)e^{-2\lambda h_j} & \frac{1}{2}\left(1+\frac{\sigma_j}{\sigma_{j+1}}\right)e^{-2\lambda h_j}\end{pmatrix}. \tag{25}$$

When $j=s+1$, we have

$$\begin{pmatrix}\widetilde{D}_{s+1}\\ \widetilde{U}_{s+1}\end{pmatrix} = e^{\lambda(\sum_{i=s+1}^{N-1}h_i)}\prod_{i=s+1}^{N-1}\mathbf{\Gamma}_{i+1}\begin{pmatrix}\widetilde{D}_N\\ 0\end{pmatrix}. \tag{26}$$

Conversely, if the observation point is in the source region ($1 \le j \le s$), $\widetilde{D}_{j+1}$ and $\widetilde{U}_{j+1}$ in layer $j+1$ can be written in terms of $\widetilde{D}_j$ and $\widetilde{U}_j$ in upper layer $j$ as

$$\begin{pmatrix}\widetilde{D}_{j+1}\\ \widetilde{U}_{j+1}\end{pmatrix} = e^{\lambda h_j}\mathbf{\Theta}_{j+1}\begin{pmatrix}\widetilde{D}_j\\ \widetilde{U}_j\end{pmatrix}, \tag{27}$$

where the propagator matrix is

$$\mathbf{\Theta}_{j+1} = \begin{pmatrix}\frac{1}{2}\left(1+\frac{\sigma_{j+1}}{\sigma_j}\right)e^{-2\lambda h_j} & \frac{1}{2}\left(1-\frac{\sigma_{j+1}}{\sigma_j}\right)\\ \frac{1}{2}\left(1-\frac{\sigma_{j+1}}{\sigma_j}\right)e^{-2\lambda h_j} & \frac{1}{2}\left(1+\frac{\sigma_{j+1}}{\sigma_j}\right)\end{pmatrix} \tag{28}$$

In layer $j = s$, we have

$$\begin{pmatrix}\widetilde{D}_s\\ \widetilde{U}_s\end{pmatrix} = e^{\lambda(\sum_{i=1}^{s-1}h_i)}\prod_{i=s-1}^{1}\mathbf{\Theta}_{i+1}\begin{pmatrix}\widetilde{D}_1\\ \widetilde{U}_1\end{pmatrix}. \tag{29}$$

At the interface of layer $s$ and $s+1$, the boundary conditions in equations (8) and (9) result in

$$\begin{pmatrix}\widetilde{D}_s\\ \widetilde{D}_s\end{pmatrix} = e^{\lambda h_s}\mathbf{\Gamma}_{s+1}\begin{pmatrix}\widetilde{D}_{s+1}\\ \widetilde{U}_{s+1}\end{pmatrix} - e^{\lambda h_s}\frac{I}{4\pi\lambda}\begin{pmatrix}1\\ e^{-2\lambda h_s}\end{pmatrix}. \tag{30}$$

Substituting equations (26) and (29) into (30) and using

$$\widetilde{U}_1 = -\widetilde{D}_1\,, \tag{31}$$

we obtain

$$\widetilde{D}_N = \frac{I}{4\pi\lambda}e^{-\lambda(z_{N-1}-z_s)}\frac{1-\frac{Q_{11}-Q_{12}}{Q_{21}-Q_{22}}e^{-2\lambda h_s}}{P_{11}-\frac{Q_{11}-Q_{12}}{Q_{21}-Q_{22}}P_{21}} \tag{32}$$

and

$$\widetilde{D}_1 = \frac{I}{4\pi\lambda}e^{-\lambda z_s}\frac{\dfrac{\left(1-\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{11}+\left(1+\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{21}}{\left(1+\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{11}+\left(1-\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{21}}-1}{\left(Q_{21}-Q_{22}\right)-\left(Q_{11}-Q_{12}\right)e^{-2\lambda h_s}\dfrac{\left(1-\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{11}+\left(1+\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{21}}{\left(1+\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{11}+\left(1-\frac{\sigma_s}{\sigma_{s+1}}\right)P'_{21}}}, \tag{33}$$

where all the elements $P_{ij}$, $P'_{ij}$, and $Q_{ij}$ ($i, j = 1,2$) are determined by

$$\begin{pmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{pmatrix} = \prod_{i=s}^{N-1}\mathbf{\Gamma}_{i+1} \tag{34}$$

$$\begin{pmatrix}P'_{11} & P'_{12}\\ P'_{21} & P'_{22}\end{pmatrix} = \prod_{i=s+1}^{N-1}\mathbf{\Gamma}_{i+1} \tag{35}$$

and

$$\begin{pmatrix}Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{pmatrix} = \prod_{i=s-1}^{1}\mathbf{\Theta}_{i+1}. \tag{36}$$

Once $\widetilde{D}_1, \widetilde{U}_1, \widetilde{D}_N$ have been determined, $\widetilde{D}_j$ and $\widetilde{U}_j$ in any layer can be obtained through equations (24) and (27), depending upon where the observation is.

To summarize, the magnetic field due to a semi-infinite source in a 1D earth can be computed by

$$\widetilde{H}_j(\lambda,z) = \frac{I}{2\pi\lambda}+\widetilde{D}_j e^{-\lambda(z-z_{j-1})}+\widetilde{U}_j e^{\lambda(z-z_{j-1})} \qquad (1 \le j \le s) \tag{37}$$

or

$$\widetilde{H}_j(\lambda,z) = \widetilde{D}_j e^{-\lambda(z-z_{j-1})}+\widetilde{U}_j e^{\lambda(z-z_{j-1})} \qquad (s+1 \le j \le N)\,. \tag{38}$$

**Computation of the Electric Field in a 1D Earth**

From equations (10) and (11), we can readily obtain the radial and vertical components of electric field in the wavenumber domain by

$$\widetilde{E}^r_j(\lambda,z) = \frac{\lambda}{\sigma_j}\left[\widetilde{D}_j e^{-\lambda(z-z_{j-1})}-\widetilde{U}_j e^{\lambda(z-z_{j-1})}\right] \tag{39}$$

$$\widetilde{E}^z_j(\lambda,z) = \frac{\lambda}{\sigma_j}\left[\widetilde{D}_j e^{-\lambda(z-z_{j-1})}+\widetilde{U}_j e^{\lambda(z-z_{j-1})}\right]. \tag{40}$$

Both magnetic and electric fields in the wavenumber domain are transformed back to the spatial domain through the Hankel transform in equation (5). We use the digital filter by Christensen (1990) to perform the Hankel transform.

## ANALYSIS

As shown in Figure 1, the total $B$ field is a linear superposition of the magnetic field due to the wire current ($AOC$), $B^w$, and to the earth current, i.e., the field $B^{1d}$. As mentioned before, there is only an azimuthal component because of the symmetry of the problem. For the following, we assumed that the magnetic permeability $\mu$ in each layer is equal to that in the air. Two special cases are of interest.

### A Uniform Half-space

Consider Figure 2 in which a current source $I$ is at a depth $h$. To model this we return to Figure 1 and the work done above. $B^w$ can be obtained by a simple Biot-Savart integral over the semi-infinite wire from $A$ to $C$,

$$B^w(r,z) = \frac{\mu I}{4\pi r}\left[1 - \frac{z-h}{\sqrt{r^2+(z-h)^2}}\right] , \qquad (41)$$

where $h$ is the depth of the buried point source, and $r$ and $z$ are the radial and vertical coordinates of the observation point in a cylindrical coordinate system. The total $B$ field has been derived in equation (23). Therefore, the field due to the earth current is

$$B^{1d}(r,z) = B(r,z) - B^w(r,z) = \frac{\mu I}{4\pi r}\left[1 - \frac{z+h}{\sqrt{r^2+(z+h)^2}}\right] . \qquad (42)$$

This expression proves the validity of the equivalent current filament model, as proposed by Nabighian et al. (1984). According to the filament model, the magnetic field on and below the earth's surface, due to a current source of strength $I$ buried at depth $h$, is exactly equivalent to the magnetic field generated by a current filament of strength $I$ which flows downward from infinity and terminates at a height $h$ above the surface (see Figure 2). Equation (42) is also consistent with the expression (3) shown by Veitch et al. (1990) using the image method.

When the buried depth $h = 0$, and the observation point is on the surface of the earth, i.e., $z = 0$, equation (42) is simplified as

$$B^{1d}(r,0) = \frac{\mu I}{4\pi r} . \qquad (43)$$

This is a very important point for surface MMR. It shows that the magnetic field is insensitive to the conductivity beneath the surface, and only related to the geometry between the source electrode and receiver. This is also true for the downhole MMR in a uniform half-space, as seen from equation (42).

### A Two-layered Earth

Assume the electrode is located at the interface of $\sigma_1$ and $\sigma_2$, i.e., $h = h_1$. The existence of a conductivity interface changes the pattern of current flow, and consequently changes the magnetic field. To quantify the difference from the half-space case, the field $B^{1d}$ can be split into two parts: one due to the primary current flow in a uniform half-space (conductivity $\sigma_1$), denoted by $B_p^{1d}$, and the other due to the secondary (or perturbed) current flow, denoted by $B_s^{1d}$, i.e.,

$$B^{1d}(r,z) = B_p^{1d}(r,z) + B_s^{1d}(r,z) . \qquad (44)$$

As shown in equation (15), generally we may not obtain a closed form for the magnetic field in the spatial domain; instead, we discuss it directly in the wavenumber domain where this field is represented with a tilde ($\widetilde{B}$). Since we have the total $\widetilde{B}(\lambda,z)$ (equation (15)), the wire $\widetilde{B}^w$ (transformed from equation (41)), and the primary $B_p^{1d}$ (transformed from equation (42)), the secondary $B_s^{1d}$ in layer 1 can be obtained from

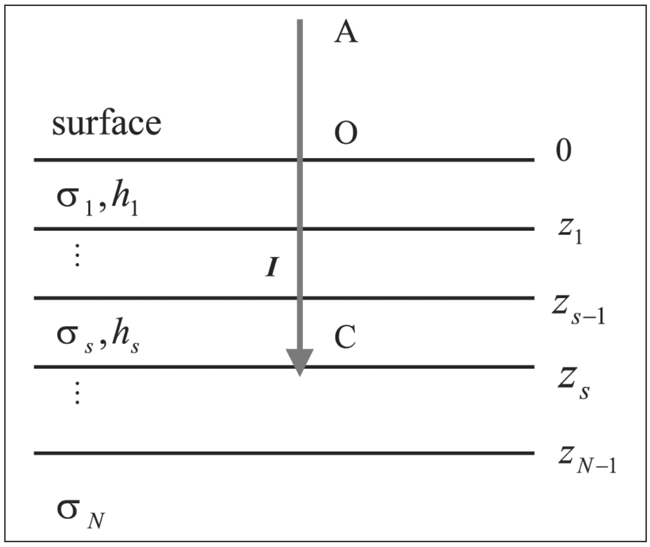


**Fig. 1. A schematic of a semi-infinite wire source terminating at an interface in a *N*-layered earth.**

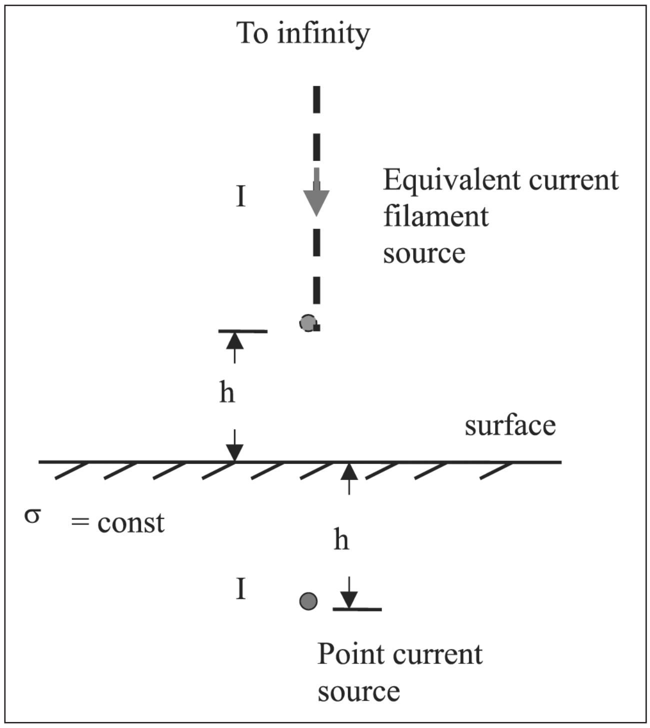


**Fig. 2. A point current source buried at a depth *h*, in a uniform half-space, and its equivalent current filament.**

$$\widetilde{B}_s^{1d}(\lambda,z) = \frac{I}{4\pi\lambda}\frac{\left(1-\frac{\sigma_2}{\sigma_1}\right)\left(1+e^{-2\lambda h_1}\right)}{\left(1+\frac{\sigma_2}{\sigma_1}\right)-\left(1-\frac{\sigma_2}{\sigma_1}\right)e^{-2\lambda h_1}}\left[e^{-\lambda(z+h_1)} - e^{-\lambda(h_1-z)}\right] . \qquad (45)$$

This expression shows that the secondary field $B_s^{1d}$ depends upon the conductivity ratio ($\sigma_2/\sigma_1$), as well as other geometric factors. When the measurement is made on the surface ($z = 0$), however, the last two exponential terms in equation (45) cancel each other, and the secondary field is null. This is a fundamental difference between the downhole MMR and surface MMR methods.

To better understand this point, we compute the current density distribution through formula (39) and (40) and then transform the result to the spatial domain. The source electrode is located on the surface, and the depth of the first layer is 50 m. The conductivity contrast between layer 2 and layer 1 is 10. The maps of the total, primary, and secondary current densities on a cross-section are plotted in Figure 3a, 3b, and 3c, respectively. The associated total, primary, and secondary $B_x$ are also shown in Figure 3d, 3e, and 3f. We note that $B_z$ is zero since all of the magnetic fields are azimuthal. For this particular cross-section, and coordinate system, $B_y$ is also zero.

As seen from Figure 3c, the secondary current is axially symmetric around the vertical coordinate axis passing through the source, so it depends only on $r$ and $z$. Edwards and Nabighian (1991) refer to this as a *poloidal* current and this figure is a numerical substantiation of their sketch of the secondary currents. Because of the symmetry of the fields, we can apply Ampère's law on a horizontal plane at depth to get

$$\oint_l B_s^{1d} dl = \mu \int_s \mathbf{J}_s^{1d} \cdot \hat{\mathbf{z}} ds = \mu I_s^{1d}, \quad \text{or} \quad B_s^{1d} = \frac{\mu I_s^{1d}}{2\pi r}, \tag{46}$$

where $I_s^{1d}$ is the current strength enclosed in the circle of radius $r$ centred on the axis. Clearly, $I_s^{1d}$ has some value beneath the surface and $B_s^{1d}$ is definitely not zero, as seen in Figure 3f. When measuring on the surface or above, $I_s^{1d} = 0$, and $B_s^{1d} = 0$. This explains, in another way, why we end up with no response from a 1D conductivity variation when making measurement at the surface of a layered earth.

To provide further physical insight, a companion figure to Figure 3 is added to demonstrate the behaviour of $\mathbf{J}^{1d}$ and $\mathbf{B}^{1d}$ in vector plots for a conductive overburden. In this case, the conductivity contrast between layer 1 and layer 2, i.e., $\sigma_1/\sigma_2$, is 10. As shown in Figure 4, the primary current density and primary $B_x$ are identical to Figure 3b and 3e. For the secondary current density and field, they just flip the signs, compared to those in Figure 3. Consequently, the total current density and $B_x$ field (Figure 4a and 4d) change dramatically.

**Borehole Measurements**

To obtain insight about the magnetic fields in a borehole, we use the example discussed by Veitch et al. (1990). As shown in the inset in Figure 5, a single buried transmitting electrode is located at a depth of 25 m, and the receivers are in a vertical borehole that is 50 m away from the source electrode. The conductivity contrast ($\sigma_1/\sigma_2$) is varied from 0.2 to 100. The azimuthal component of the magnetic field clearly shows the position of the conductivity boundary. This is regarded as one of the advantages for downhole MMR compared to electric potential profiles (Veitch et al., 1990). In addition, this plot is almost identical to Veitch et al.'s Figure 7a, and helps to verify the solution derived in this note.

In addition, when we fix $\sigma_1/\sigma_2 = 10$, but only change the depth $d$ of the source electrode, varying from 0 (at the surface), going down through the conductive overburden, and to 200 m in the resistivity basement, we have magnetic field distributions shown in

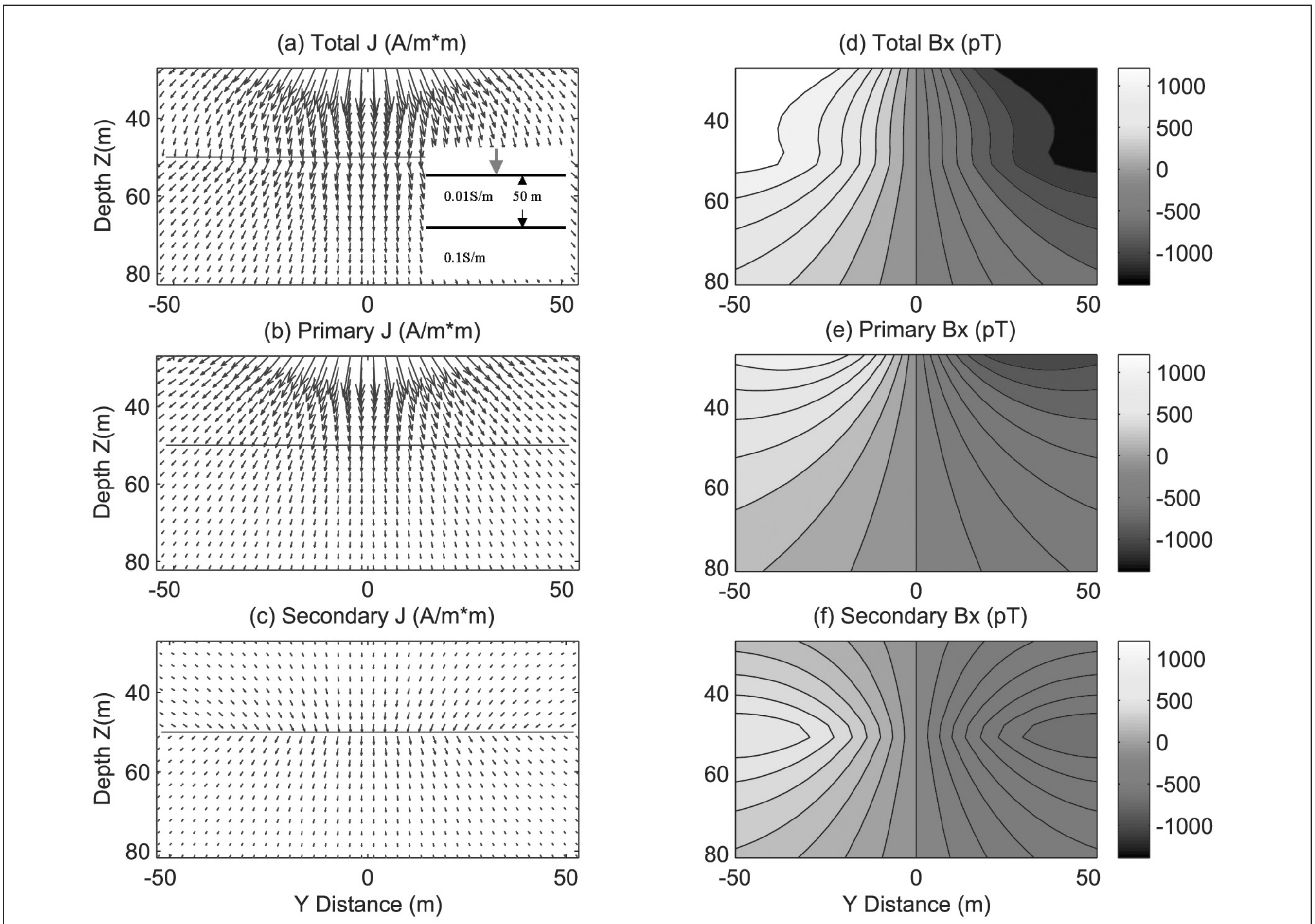


**Fig. 3. Current density and magnetic field $B_x$ distributions over a two-layered earth. The inset is the 1D model used. The total current is shown in (a). This can be decomposed into a primary current of a uniform half-space and a secondary current due to the presence of the conductivity interface at depth 50 m. These are shown in (b) and (c) respectively. The corresponding total, primary, and secondary $B_x$ are in (d), (e), and (f). The vertical component of B is zero everywhere inside the earth.**

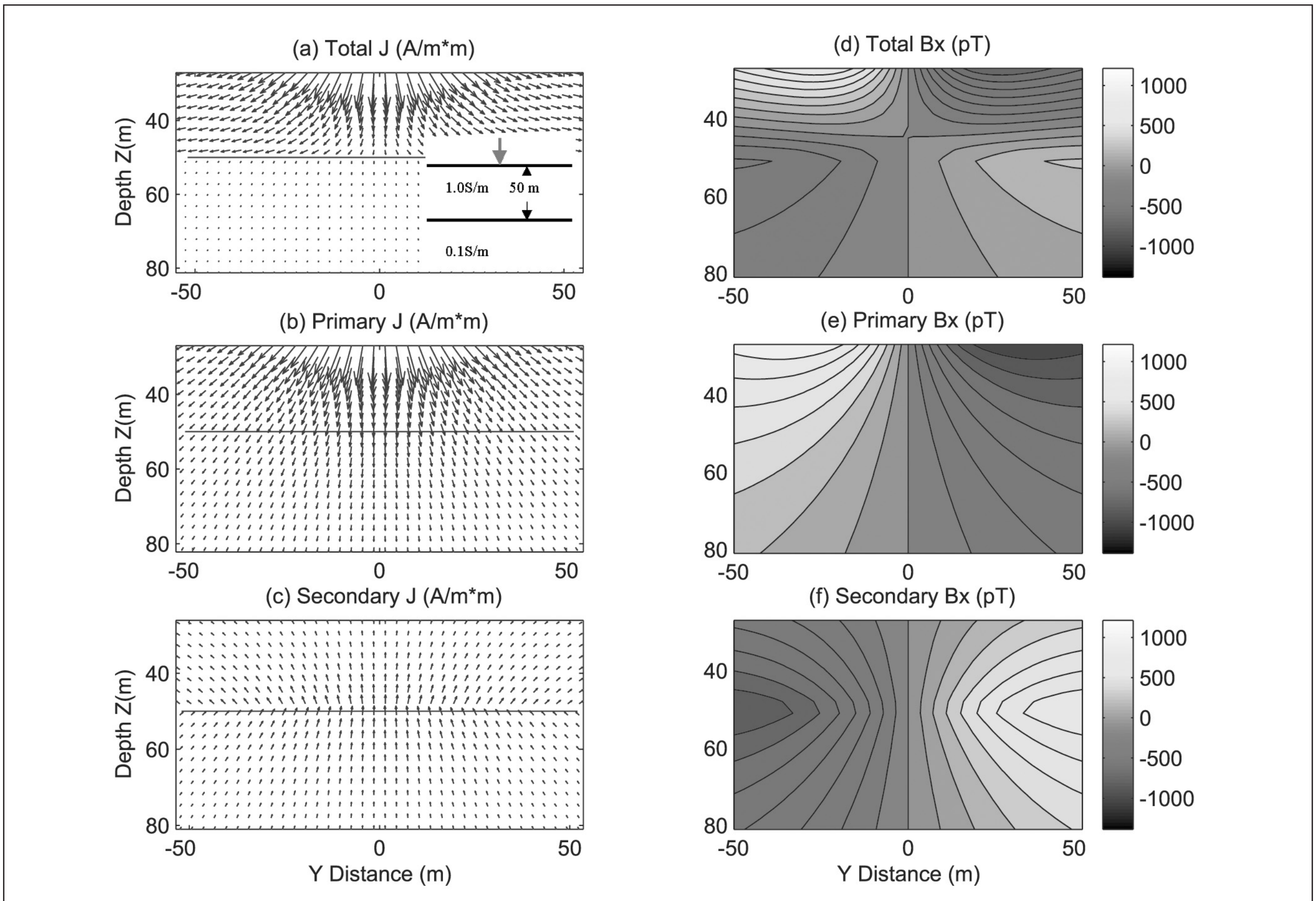


**Fig. 4. Current density and magnetic field $B_x$ distributions over a two-layered earth with a conductive overburden. The conductivity contrast $\sigma_1$ / $\sigma_2$ is 10.**

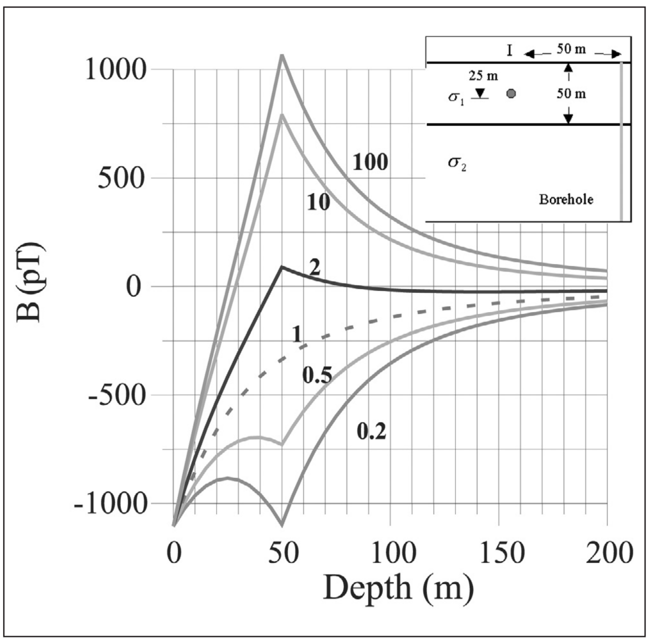


**Fig. 5. The magnetic field $B^{1d}$ variation versus conductivity contrast $\sigma_1$ / $\sigma_2$ for a two-layer earth. The inset is the 1D model used, and the conductivity contrast $\sigma_1$ / $\sigma_2$ is labelled on the respective profile.**

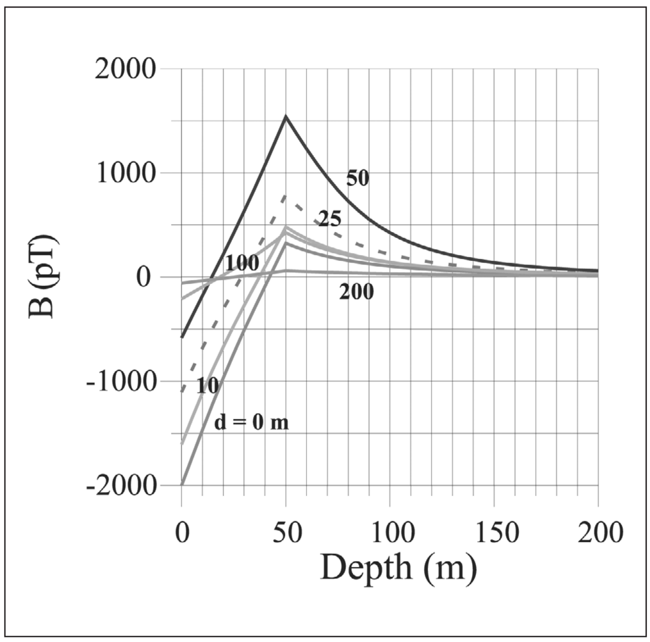


**Fig. 6. The magnetic field $B^{1d}$ variation versus depth of the source electrode for the two-layer earth shown in Figure 5. The conductivity contrast $\sigma_1$ / $\sigma_2$ is fixed at 10. Depth $d$ is labelled on the respective profile.**

Figure 6. All these results are in an excellent agreement with those in Figure 6a presented by Veitch et al.

## CONCLUSION

We have derived a general solution for the magnetic and electrical fields due to a point source in a layered earth. This solution is relevant to interpreting downhole and marine MMR data, in which 1D inversion and calculation of apparent resistivity soundings are required. The solution also provides a theoretical base from which to understand certain unique behaviours of MMR data.

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