Nonlinear Inversion for Multiple Objects in Transient Electromagnetic Induction Sensing of Unexploded Ordnance: Technique and Applications

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Abstract-We develop an inversion technique to process overlapping data that arise from closely spaced targets. In contrast to a usual single-object inversion model, a multiobject problem is more challenging because of the increased number of parameters to be found and because of the additional nonlinearity and nonuniqueness. Our solution strategy is to break down the full problem into a sequence of smaller problems so that optimization is conducted in a lower dimensional model space. In the numerical implementation, a set of nonlinear model parameters, e.g., the locations of the underlying sources, is sought while the set of linear model parameters, i.e., their polarization tensors, are updated accordingly in a nested manner. This is an explicit separable nonlinear optimization technique that we cast. We employ a joint diagonalization to find an average principal direction among multiple magnetic polarizability tensors. Since the principal directions are more sensitive to the inaccuracies in the estimated polarization tensor, we suggest a subsequent procedure to optimize the two sets of parameters: orientation and principal polarizations of objects. For initialization, we propose a selected multistart nonlinear algorithm for source localizations that paves an efficient way to find a good initial guess of model parameters and makes the nonlinear inversion effectively automated. We report the new applications of the technique to the test-stand and field data acquired with next-generation sensor systems of the TEMTADS and MetalMapper and study the issue of the spatial resolution of overlapping anomalies through inversions and using the metric defined as the total uncertainty of the polarizabilities.

Index Terms—Electromagnetic induction (EMI), multiple objects, nonlinear inversion, transient response, unexploded ord-nance (UXO).

I. INTRODUCTION

I N electromagnetic induction (EMI) sensing of unexplodedordnance (UXO)-contaminated sites, data inversion of target signatures (e.g., dipolar polarizabilities) plays a central part in the signal processing for its aim to provide accurate inputs for

Manuscript received August 3, 2010; revised December 23, 2010 and February 17, 2011; accepted February 20, 2011. Date of publication May 11, 2011; date of current version September 28, 2011. This work was supported by a Strategic Environmental Research and Development Program Grant MM-1637.

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Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TGRS.2011.2132138

discrimination of UXO from metallic debris [1]–[24] in both time and frequency domains. It is commonly assumed that EMI sensors sense a nearby object, and thus, such inverse problems are preceded with a single-object parametric model. In practice, this is not always the case. In sites where subsurface munitions are distributed closely in space, EMI sensors can record overlapping signals from multiple objects. The processing of data from multiple objects has been a practical issue and has received growing attention in the EMI community.

For example, Hu and Collins [25] adopted the well-known blind source separation (e.g., independent component analysis [26]) technique and attempted to separate unknown dipolar signatures of each object in the frequency domain and to identify the objects by comparing the extracted source signatures with those in a library. Bell [27] attempted to invert transient overlapping signals for parameters of equivalent dipoles. Shubitidze et al. [28] applied their normalized surface magnetic charge model to process overlapping signals using frequencydomain experimental EMI data. Grzegorczyk et al. [29] demonstrated a Newton method to detect multiple objects using numerical and experimental time-domain data. In [30], we developed an inversion technique that attempts to recover parameters for each object from overlapping data. We examined our multiobject inversion technique using Geonics-EM63 simulated and field data collected at Camp Sibert, Albama. The blind tests of our proposed methodology using the EM63 field data were encouraging and demonstrated that it was possible to invert for multiple target signatures from overlapping signals if data quality was adequate.

In this paper, we extend to our previous presentation [30] and evaluate its practical applicability. For the fact that most transient experimental and field measurements are widely used, our development is formulated with time-domain data. The technique is, in principle, easily adapted to frequency-domain data. In Section II, the necessary forward modeling equations are presented. In Section III, we cast the multiobject inversion problem as a separable nonlinear optimization problem and implement a selected multistart nonlinear search for source localizations. A joint diagonalization method is then used to find an average principal direction of each object. In Section IV, we apply the technique to test-stand and field data recorded by the next-generation sensor array systems of TEMTADS [32] and MetalMapper [33]. Extensive inversions are carried out to provide insight about the practicalities of resolving closely spaced objects. Our results, using field data, demonstrate that the technique is practical.

II. PHYSICAL MODEL AND THE PROBLEM FORMULATION

Consider a standard EMI system consisting of a transmitting coil and a receiving coil which may be colocated or not. In active sensing, a primary field emitted from a transmitter illuminates a nearby object and its changes with time induce eddy currents in the metal object. These induced currents produce a transient secondary magnetic field that is measured by a receiver. In practice, an array of sensors is generally positioned above the surface. For a UXO survey where the target dimension is often small relative to the target-sensor distance, the primary fields around the target can be approximately uniform and the induced eddy currents in the target are localized and dominantly produce dipole responses [34], [35]. As a leading order approximation, the EMI nature of a metal target can be well represented by an equivalent induced dipole. Here, we assume that dipole location and orientation are independent of time. Let the target location be denoted by r; then, the induced dipole moment $\mathbf{m}(t)$ at time t is represented by [1]–[15]

$$\mathbf{m}(t) = P(t)\mathbf{B}_T(\mathbf{r}, \mathbf{r}_{Tx}) \tag{1}$$

where \mathbf{B}_T is the exciting magnetic field vector at the target location from a transmitter at \mathbf{r}_{Tx} and P(t) is a 3 × 3 symmetric magnetic polarizability tensor (MPT)

$$P(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) & p_{13}(t) \\ p_{12}(t) & p_{22}(t) & p_{23}(t) \\ p_{13}(t) & p_{23}(t) & p_{33}(t) \end{bmatrix}.$$
 (2)

Physically, the element $p_{ij}(t)$ of the tensor P(t) represents a dipole component in the *i*th Cartesian direction due to a primary field in *j*th Cartesian direction. This polarizability tensor P(t) has an eigendecomposition

$$P(t) = \sum_{i=1}^{3} L_i(t) \mathbf{e}_i \mathbf{e}_i^{\mathrm{T}}$$
(3)

where \mathbf{e}_i (i = 1, 2, 3) is the orthonormal eigenvector representing the *i*th principal direction of dipolar polarization with respect to a reference system and $L_i(t)$ is the principal polarization strength that is a function of the geometry and material of a target. In other words, P(t) contains the information regarding the geometry and material of a target as well as its orientation. In the time domain, the P(t) in (1) includes a convolution with a transmitter waveform I(t) [24], i.e., $P(t) = P_s(t) * I'(t)$, where $P_s(t)$ is an impulse response and ' denotes a derivative operation. For an abrupt switch-off excitation, P(t) might be viewed approximately as an impulse response tensor. For most off-time measurements, we usually do not need to deconvolve this tensor but rather directly use it in practice (e.g., see [4], [5], [8], [10], and [13]–[15]).

At the observation location \mathbf{r}_{Rx_i} , the magnetic vector potential \mathcal{A} due to a moment $\mathbf{m}(t)$ is [35]

$$\mathcal{A}(t) = \frac{\mu_0}{4\pi} \frac{\mathbf{m}(t) \times (\mathbf{r}_{Rx_i} - \mathbf{r})}{|\mathbf{r}_{Rx_i} - \mathbf{r}|^3}.$$
 (4)

For a finite size of a receiver loop, we measure voltage that is expressed as

$$d\left(\mathbf{r}_{Rx_{i}},t\right) = \int_{l_{Rx_{i}}} \mathcal{A} \cdot d\mathbf{l} = \frac{\mu_{0}}{4\pi} \int_{l_{Rx_{i}}} \frac{d\mathbf{l} \times (\mathbf{r} - \mathbf{r}_{Rx_{i}})}{|\mathbf{r} - \mathbf{r}_{Rx_{i}}|^{3}} \cdot \mathbf{m}(t).$$
(5)

According to the Biot–Savart law [35], the line integral along l_{Rx_i} in (5) is a magnetic induction field vector at dipole location **r** produced by a receiver loop with unit current and may be denoted as

$$\mathbf{B}_{R}\left(\mathbf{r},\mathbf{r}_{Rx_{i}}\right) = \frac{\mu_{0}}{4\pi} \int_{l_{Rx_{i}}} \frac{d\mathbf{l} \times (\mathbf{r} - \mathbf{r}_{Rx_{i}})}{\left|\mathbf{r} - \mathbf{r}_{Rx_{i}}\right|^{3}}.$$
 (6)

Substituting (1) into (5) and using (6), we have the secondary response d_i compactly written as [1], [10], [13]

$$d_i\left(\mathbf{r}_{Rx_i}, t\right) = \mathbf{B}_R^{\mathrm{T}}\left(\mathbf{r}, \mathbf{r}_{Rx_i}\right) P(t) \mathbf{B}_T\left(\mathbf{r}, \mathbf{r}_{Tx_i}\right)$$
(7)

where superscript T denotes a transpose and subscript i denotes the *i*th measurement in M sensing locations. The derivation of (7) illustrates the principle of reciprocity [1] and that an induced dipole response in the receiver coil can be calculated as a scalar product of \mathbf{B}_R and \mathbf{m} . Equation (7) describes the basic EMI process of illuminating, scattering, and sensing.

For the inversion development, we rearrange (7) as

$$d_i \left(\mathbf{r}_{Rx_i}, t \right) = \mathbf{a}_i^{\mathrm{T}} \left(\mathbf{r}, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i} \right) \mathbf{q}(t) \tag{8}$$

where $\mathbf{a}_i(\mathbf{r}, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i})$ is a 6 × 1 column vector representing spatial sensitivities of the *i*th sensor to the object located at \mathbf{r} and $\mathbf{q}(t)$ is a 6 × 1 column vector whose components are the elements of the polarizability tensor P(t) of an object. They are given by

$$\mathbf{a}_{i}\left(\mathbf{r},\mathbf{r}_{Rx_{i}},\mathbf{r}_{Tx_{i}}\right) = \begin{bmatrix} B_{R}^{x}B_{T}^{x} \\ B_{R}^{x}B_{T}^{y} + B_{R}^{y}B_{T}^{x} \\ B_{R}^{x}B_{T}^{z} + B_{R}^{z}B_{T}^{x} \\ B_{R}^{y}B_{T}^{y} \\ B_{R}^{y}B_{T}^{z} + B_{R}^{z}B_{T}^{y} \\ B_{R}^{z}B_{T}^{z} + B_{R}^{z}B_{T}^{z} \end{bmatrix} \qquad \mathbf{q}(t) = \begin{bmatrix} p_{11}(t) \\ p_{12}(t) \\ p_{13}(t) \\ p_{22}(t) \\ p_{23}(t) \\ p_{33}(t) \end{bmatrix}$$
(9)

where $[B_R^x \ B_R^y \ B_R^z]^T$ and $[B_T^x \ B_T^y \ B_T^z]^T$ are the Cartesian components of field vectors $\mathbf{B}_R(\mathbf{r}, \mathbf{r}_{Rx_i})$ and $\mathbf{B}_T(\mathbf{r}, \mathbf{r}_{Tx_i})$.

Alternatively, by substituting (3) into (7), we obtain another scalar-product form of the secondary response

$$d_i\left(\mathbf{r}_{Rx_i}, t\right) = \mathbf{g}_i^{\mathrm{T}}\left(\mathbf{r}, \xi, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i}\right) \mathbf{f}(t).$$
(10)

In (10), ξ denotes one set of Euler angles (ϕ, θ, ψ) that describes the orientation of a dipolar object and hence determines the Euler vectors \mathbf{e}_i and

$$\mathbf{g}_{i}^{\mathrm{T}}(\mathbf{r},\xi,\mathbf{r}_{Rx_{i}},\mathbf{r}_{Tx_{i}}) = \begin{bmatrix} B_{R}^{\mathrm{T}}\mathbf{e}_{1}\mathbf{e}_{1}^{\mathrm{T}}B_{T} & B_{R}^{\mathrm{T}}\mathbf{e}_{2}\mathbf{e}_{2}^{\mathrm{T}}B_{T} \\ B_{R}^{\mathrm{T}}\mathbf{e}_{3}\mathbf{e}_{3}^{\mathrm{T}}B_{T} \end{bmatrix}$$
$$\mathbf{f}^{\mathrm{T}}(t) = \begin{bmatrix} L_{1}(t) & L_{2}(t) & L_{3}(t) \end{bmatrix}.$$
(11)

Consider η multiple objects at locations of $\mathbf{r}_1, \ldots, \mathbf{r}_\eta$ with orientations ξ_1, \ldots, ξ_η and characterized with polarizations $\mathbf{q}_1(t), \ldots, \mathbf{q}_\eta(t)$ or $\mathbf{f}_1(t), \ldots, \mathbf{f}_\eta(t)$, each of which is defined in (9) and (11). In our following work, we assume that the electromagnetic interaction between the objects can be neglected. Although this does not hold always [6], [31], our previous work [30] and works by others [25]–[29] show that this assumption may often be valid in practice. By neglecting interaction and thus using linear superposition of individual contributions, we have $d_i(\mathbf{r}_{Rx_i}, t) = \sum_{k=1}^{\eta} \mathbf{a}_i^{\mathrm{T}}(\mathbf{r}_k, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i}) \mathbf{q}_k(t) = \sum_{k=1}^{\eta} \mathbf{g}_i^{\mathrm{T}}(\mathbf{r}_k, \xi_k, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i}) \mathbf{f}_k(t)$, where $\mathbf{a}_i(\mathbf{r}_k, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i})$ and $\mathbf{g}_i(\mathbf{r}_k, \xi_k, \mathbf{r}_{Rx_i}, \mathbf{r}_{Tx_i})$, defined in (9) and (11), are the spatial sensitivities of the *i*th sensor to the *k*th object located at \mathbf{r}_k with orientation ξ_k .

To simplify the notation, we now suppress the position vectors of the sensor coils. It is understood that the sensor information is a subscript indexed in the recordings and in the sensitivity vectors. The measurements made at M sensing locations, in the presence of noise, can be conveniently expressed in a vector-matrix notation

$$\mathbf{d}(t) = \sum_{k=1}^{\eta} A(\mathbf{r}_k) \mathbf{q}_k(t) + \mathbf{n}(t)$$
(12)

or

$$\mathbf{d}(t) = \sum_{k=1}^{\eta} G(\mathbf{r}_k, \xi_k) \mathbf{f}_k(t) + \mathbf{n}(t)$$
(13)

where $\mathbf{d}(t) = [d_1(t), \ldots, d_M(t)]^{\mathrm{T}}$ is an $M \times 1$ measured data vector at time t, $\mathbf{n}(t)$ is the additive noise vector, and $A(\mathbf{r}_k)$ is an $M \times 6$ matrix denoting the sensitivities of the M sensors to the kth object located at \mathbf{r}_k . Its transpose is given by $A^{\mathrm{T}}(\mathbf{r}_k) =$ $[\mathbf{a}_1(\mathbf{r}_k) \ldots \mathbf{a}_M(\mathbf{r}_k)]$. $G(\mathbf{r}_k, \xi_k)$ is an $M \times 3$ matrix denoting the sensitivities of the M sensors to the kth object located at \mathbf{r}_k and oriented at ξ_k . Its transpose is given by $G^{\mathrm{T}}(\mathbf{r}_k, \xi_k) =$ $[\mathbf{g}_1(\mathbf{r}_k, \xi_k) \ldots \mathbf{g}_M(\mathbf{r}_k, \xi_k)]$.

The observed EMI response, as illustrated in (12) and (13), is a linear combination of the finite dipole polarizations of multiple objects with spatial weighting coefficients that depend upon the locations and/or orientations of objects. Both formulations are the generic dipole-based expressions for modeling and inversion of EMI anomalies and are used interchangeably in our development.

III. INVERSION TECHNIQUES

In a transient electromagnetic (TEM) system, we usually have measurements of $D = [\mathbf{d}(t_1), \dots, \mathbf{d}(t_N)]$ at a series of time instants t_1, \dots, t_N . Given space-time data D and provided that the number η of buried objects is known or assumed, the goal of our inverse problem is to determine the locations, orientations, and principal transient polarizations, i.e., $[\mathbf{r}_1, \xi_1, \dots, \mathbf{r}_{\eta}, \xi_{\eta}, \mathbf{f}_1(t_j), \dots, \mathbf{f}_{\eta}(t_j)], j = 1, \dots, N$, of the multiple objects that best explain the spatial-temporal data.

The inversion of a data set that includes multiple objects is conceptually simple, but numerically nontrivial. It involves more parameters to be solved for, and thus, the nonlinearity and nonuniqueness of the inverse problem is greatly increased. In this high-dimensional model space, the choice of an initial guess becomes critical.

To tackle these numerical challenges, we use the fact that the EMI response is linear with respect to dipolar polarization and nonlinear with respect to the locations and/or orientations of objects. In [30], we presented a solution strategy (similar to that in [36] used for solving for a single-object case) that decomposed the inverse problem into several steps, in each of which one major set of model parameters is sought. Briefly, we first find an estimate for the nonlinear location parameters and their associated linear polarization parameters. The orientation of each object is then extracted from the polarizability tensors through an eigendecomposition in (3). These steps as well as improvements to our earlier work [30] are elaborated in the following.

A. Extracting Source Locations and Polarization Tensors

We first group our model parameters into two parts: a nonlinear part consisting of source locations $\mathbf{r} = \text{vec}[\mathbf{r}_1, \dots, \mathbf{r}_\eta]$ and a linear part consisting of source polarizations $\mathbf{v}(t) =$ $\text{vec}[\mathbf{q}_1(t), \dots, \mathbf{q}_\eta(t)]$ at time instant t. Here, $\text{vec}[\cdot]$ represents a vectorization operation, i.e., stacking all vectors into a single column. Referring to (12), we say that both parameter sets are separable since the matrix A is independent of source polarizations $\mathbf{q}_k(t)$. Using this separable property, we treat finding source locations as a primary step and estimating polarizations as an intermediate step.

1) Finding Multistart Points: To start the inversion, we need an initial guess of the locations of η objects. This might be done by examining the EMI responses and using the centroid of each anomaly peak as an initial horizontal location for a causative source. In practice, however, the spatial pattern of the response can vary significantly depending upon the orientation of the object and the transmitter–receiver configuration, e.g., the data may show a two-peak anomaly for a single object or just a one-peak anomaly for two closely spaced objects [15], [30]. Thus, this technique cannot be guaranteed to yield good initial locations. This has previously been discussed in [30], and it will be seen again here when we work with some new-generation sensor systems.

We propose the following algorithm to find starting guesses for a nonlinear inversion. First, define a region of interest (ROI) that is sufficiently large to cover the active objects. In this initial sampling process, one might use a uniform grid and create potential combined locations in the ROI. However, we consider that this way can be unpractical since it can produce an undesirably large number of possible source configurations, e.g., 125^2 , to be evaluated even for a two-object case in a coarse grid of $5 \times 5 \times 5$ over a volume of $1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$. Therefore, we chose to randomly create trial (typically a few hundred) locations for η objects within the ROI. Our processing experience shows that this amount of random sampling gives a good balance between computational efforts and accuracy.

For each trial set of locations, we evaluate the misfit

$$\Phi_d(\mathbf{r}) = \sum_{j=1}^N \left\| W_j \left(\mathbf{d}_{\text{obs}}(t_j) - \tilde{\mathbf{d}}(t_j) \right) \right\|^2.$$
(14)

$$\tilde{\mathbf{d}}(t_j) = \sum_{k=1}^{\eta} A(\tilde{\mathbf{r}}_k) \tilde{\mathbf{q}}_k(t_j)$$
(15)

where $\tilde{\mathbf{r}}_k$ denotes the trial locations and $\tilde{\mathbf{q}}_k$ denotes the estimated polarizations. These are computed by solving a constrained linear inverse problem

$$\tilde{\mathbf{v}}(t_j) = \arg\min_{\mathbf{v}(t_j)} \left\| \mathbf{d}_{obs}(t_j) - \sum_{k=1}^{\eta} A(\tilde{\mathbf{r}}_k) \mathbf{q}_k(t_j) \right\|^2$$

s.t.
$$p_{k,ii}(t) \ge 0$$

$$|p_{k,ij}(t)| \le \frac{1}{2} \left[p_{k,ii}(t) + p_{k,jj}(t) \right].$$
(16)

The constraints imposed in (16) arise from the fact that the principal polarizations $L_i(t)$ are physically positive in (3) and from the numerical equivalence that P(t) in (2) and (3) is a symmetric positive definite matrix [37], [38]. From the theorem in [37], [38]: A symmetric matrix is positive definite if and only if all principal minors are positive. In our case, the MPT is a 3×3 symmetric matrix, and for its positive definiteness, the corresponding seven principal minors are required to be positive: $det(P) \ge 0$, $p_{ii}p_{jj} - p_{ij}^2 \ge 0$, $p_{ii} \ge 0$, i, j = 1, 2, 3. The first two constraints are nonlinear. However, using the inequality of $(\sqrt{p}_{ii} - \sqrt{p}_{jj})^2 \ge 0$, we can convert $p_{ii}p_{jj} \ge p_{ij}^2$ into the equivalent linear inequality constraints $(p_{ii} + p_{jj}) \ge$ $2|p_{ij}|$. The nonlinear inequality constraints of det(P) is not easily handled and is excluded in our current implementation. The linear inequality constraints used in (16), which can be easily implemented using Matlab function lsqlin [39], are consequently necessary but not sufficient for a matrix to be semipositive definite. We say a "semi-" because of the equalities included in those constraints. To impose a full positive definite property on the polarization tensor, we may need further development [40].

After a forward evaluation using (14), the trial starting locations are ranked according to their misfit. A small population of points (typically a few tens) with low $\Phi_d(\mathbf{r})$, and whose locations are significantly distinct from other trial locations, are selected as multistart points for a nonlinear inversion.

2) Nonlinear Optimization: In the next step, we use the suite of starting locations and find both locations and polarizations by carrying out a nonlinear optimization. Either a Levenberg–Marquardt approach or trust region interior point method [41], [42] can be used. If the current location is \mathbf{r}_c , the updated location and polarizations are found by solving the following optimization problem:

$$\tilde{\mathbf{r}} = \arg\min_{\mathbf{r}} \sum_{j=1}^{N} \left\| W_{j} \left(\mathbf{d}_{obs}(t_{j}) - \tilde{\mathbf{d}}(\mathbf{r}_{c}, t_{j}) - J(\mathbf{r}_{c})(\mathbf{r} - \mathbf{r}_{c}) \right) \right\|^{2}$$
$$\tilde{\mathbf{d}}(\mathbf{r}_{c}, t_{j}) = \sum_{k=1}^{\eta} A(\mathbf{r}_{c,k}) \tilde{\mathbf{q}}_{k}(t_{j})$$
subject to $\|\mathbf{r} - \mathbf{r}_{c}\| < \Delta_{\mathbf{r}}$ (17)

where $J(\mathbf{r}_c)$ is the Jacobian for furnishing the descent direction in the local linearized search, $\tilde{\mathbf{d}}(\mathbf{r}_c, t_j)$ are the predicted data at \mathbf{r}_c , and $\Delta_{\mathbf{r}}$ is a positive scalar used to provide a local ball within which \mathbf{r} is allowed to change w.r.t. \mathbf{r}_c (for details about the algorithms, see [41] and [42]). Note that these local searches are conducted within the ROI. Evaluation of the predicted data requires that $\tilde{\mathbf{q}}_k$ be evaluated. This is done as before by solving the constrained least square problem (16). The iterations in (20) are continued until convergence criteria are satisfied. This yields final locations $\tilde{\mathbf{r}}_k$ and polarizations $\tilde{\mathbf{q}}_k$, $k = 1, \ldots, \eta$.

B. Determining Principal Directions and Polarizations

The previous computation has provided us with the estimated locations of each object and an MPT for each object at time $t_i, j = 1, \ldots, N$. From the estimated P(t) at time t and its eigendecomposition, the corresponding orientation angles $\xi =$ (ϕ, θ, ψ) can be obtained through their relationship with the components of the Euler vectors e_i [30]. This was the approach we used in [30]. Each time channel yields its own orientation, and generally, there is no guarantee that they are close to each other. For the dipole model assumed here, the principal polarization directions are assumed not varying with time, and hence, a better approach is to determine an average principal direction across a range of times using a joint diagonalization technique (see [43] and [44] for details). For instance, the technique can be employed to find the best orthogonal matrix \tilde{E} so that $P(t_1) = \tilde{E}L_1\tilde{E}^{\mathrm{T}}, \dots, P(t_N) = \tilde{E}L_N\tilde{E}^{\mathrm{T}}, \text{ where } L_1, \dots, L_N$ are found as 3×3 diagonal matrices as possible. In other words, E represents an "average eigenstructure" shared by the matrices P_1, \ldots, P_N . We have implemented this but have still found unsatisfactory results in a few cases. The reason can be seen from first-order perturbation analysis. For eigenvector (principal direction) perturbations $\Delta \mathbf{e}_i$ due to error in P, we have [13], [38]

$$\Delta \mathbf{e}_{i} = \sum_{i \neq k} \frac{\mathbf{e}_{k,0}^{\mathrm{T}} \Delta P \mathbf{e}_{i,0}}{L_{i,0} - L_{k,0}} \mathbf{e}_{k,0}, \qquad i, k = 1, \dots, 3$$
(18)

and this can become erroneously large even for small perturbations ΔP of the MPT if there are small differences between the principal polarizations. Considering this potential instability problem, measurement errors, and possible tradeoff among the magnetic polarizability tensors of multiple objects, we have used the following strategy. We use the orientation found by the joint diagonalization aforementioned as a starting value and then do a nonlinear update to determine optimal orientations of multiobjects by fixing their locations. In this case, formulation (13) is used and the separable parameter sets are the orientations of objects and their principal polarizations. The nonlinear optimization algorithm is analogous to that used to find the locations of the objects. Defining $\xi = \text{vec}[\xi_1, \dots, \xi_\eta]$ and from a set of starting or current orientations ξ_c of multiple objects, we obtain their optimal estimate as

$$\tilde{\xi} = \arg\min_{\xi} \sum_{j=1}^{N} \left\| W_j \left(\mathbf{d}_{obs}(t_j) - \tilde{\mathbf{d}}(\xi_c, t_j) - J(\xi_c)(\xi - \xi_c) \right) \right\|^2$$
$$\tilde{\mathbf{d}}(\xi_c, t_j) = \sum_{k=1}^{\eta} G(\tilde{\mathbf{r}}_k, \tilde{\xi}_{c,k}) \mathbf{f}_k(t_j)$$
subject to $\|\xi - \xi_c\| < \Delta_{\xi}$ (19)

where $J(\xi_c)$ is a Jacobian matrix with respect to the orientations, Δ_{ξ} is a positive scalar representing the trust region size [41], [42] and plays a similar role as in (20), and $\tilde{\mathbf{d}}(\xi_c, t_j)$ denotes the predicted data. The evaluation of $\mathbf{f}_k(t_j)$ is done similarly to finding $\mathbf{q}_k(t_j)$ in the location search. That is, we solve the linear subproblem minimizing

$$\tilde{\mathbf{u}}(t_j) = \arg\min_{\mathbf{u}(t_j)} \left\| \mathbf{d}_{\text{obs}}(t_j) - \sum_{k=1}^{\eta} G(\tilde{\mathbf{r}}_k, \tilde{\xi}_{c,k}) \mathbf{f}_k(t_j) \right\|^2$$

s.t. $L_{k,i}(t) \ge 0$ (20)

where $\mathbf{u}(t) = \text{vec}[\mathbf{f}_1(t), \dots, \mathbf{f}_\eta(t)]$. Thus, the process of nested inversions for orientation is identical to that for location. This estimate of source orientations and polarizations is chosen when there is improvement in data fit over eigendecomposition-based estimation.

With the completion of the calculations about locations, orientations, and polarizations of multiple objects, in the following, we apply the technique to test-stand and field data sets using a two-object model since this scenario appears frequently in practice.

IV. APPLICATIONS

To demonstrate our methodology, we present the results using TEMTADS and MetalMapper TEM data. These are recently developed new-generation EMI sensor systems. TEMTADS [32] is a single-component multistatic system. It consists of a horizontally arranged coplanar array of 5×5 transmitters and receivers [Fig. 1(a)] with a sensor footprint of $2 \text{ m} \times 2 \text{ m}$. The sizes of its transmitters and receivers are $35 \text{ cm} \times 35 \text{ cm}$ and $25 \text{ cm} \times 25 \text{ cm}$, respectively. It has 115 logarithmically spaced gates between 0.042 and 24.35 ms. For each transmitter excitation, TEMTADS records the response at all receivers. Thus, it has spatial-temporal data of 625×115 for a static (cued) survey.

The MetalMapper [33] is a multicomponent system consisting of three orthogonal transmitters (roughly $1 \text{ m} \times 1 \text{ m}$) and seven three-component receiver cubes of 0.1 m [Fig. 1(b)]. The sensor footprint is $1 \text{ m} \times 1 \text{ m}$. The MetalMapper has 42 logarithmical time gates ranging from 0.106 to 7.91 ms. It can be employed in two survey modes: a dynamic mapping survey for target detection and a static (cued) measurement over detected targets for use in discrimination. At one static



Fig. 1. (a) TEMTADS: A single-component multistatic system consisting of a horizontally arranged coplanar array of 5×5 (red squares) transmitters and (blue squares) receivers. Each transmitter is $35 \text{ cm} \times 35 \text{ cm}$, and each receiver is $25 \text{ cm} \times 25 \text{ cm}$. (b) MetalMapper: A multicomponent system consisting of (red squares) three orthogonal $1 \text{ m} \times 1$ m transmitters and (in blue) seven three-component receiver cubes of 0.1 m.



Fig. 2. TEMTADS multiple-object measurement configuration: (a) 4.2'' mortar + half shell (deep and big clutter) and (b) 4.2'' mortar + nose piece (shallow and small clutter). H_d denotes a horizontal separation between the two objects.

sounding, the MetalMapper can record spatial-temporal data of 63×42 . In the summer of 2009, both systems were operated in an Environmental Security Technology Certification Programsponsored classification study at the former Camp San Luis Obispo (SLO), CA.

A. TEMTADS Test-Stand Data

Test-stand data from two-object configurations, i.e., 4.2'' mortar + half shell (deep and big clutter) and 4.2'' mortar + nose piece (shallow and small clutter), are used to evaluate our methodology. Fig. 2 schematically shows the configuration in which the 4.2'' mortar is horizontally centered 61 cm below the array and is kept stationary during the experiment. The clutter, either consisting of a half shell at a depth 44 cm in Fig. 2(a) or a nose piece at a depth of 27 cm in Fig. 2(b), was moved horizontally at increments of 10 cm from 0 to 150 cm from the center of the array. A horizontal separation between the two objects is marked as H_d in the figures.

Fig. 3 shows the polarizabilities of all three items. These polarizabilities have been obtained by inverting single-object data when the 4.2" mortar, the half shell, and the nose piece were positioned separately under the array center at the depths of 61, 44, and 27 cm. We use this set of polarizabilities as our "ground truth" to compare with polarizations recovered from the overlapping data. Many inversions were carried out but the essential results can be encapsulated by looking at some easier scenarios where separations between the objects are large and other scenarios where the objects are close together. For the TEMTADS system, we choose to show the monostatic data where transmitters and receivers are colocated.



Fig. 3. Polarizabilities used in the TEMTADS test-stand experiments. (a) 4.2" mortar. (b) Half shell. (c) Nose piece.



Fig. 4. Example 1a of TEMTADS test-stand overlapping data: 4.2'' mortar + half shell with $H_d = 50$ cm. Monostatic image at $t_1 = 0.042$ ms. The white dots in the images denote the center positions of the sensors. (a) Single-object inversion. (b) Two-object inversion. On the polarization plots, red and blue dash curves represent the known polarizations of 4.2'' mortar and half shell shown in Fig. 3(a) and (b).

1) Examples 1—Large Horizontal Separation: As example 1a, we use data from two objects, a 4.2'' mortar + half shell, when their horizontal separation is $H_d = 50$ cm. This is the overlapping case from two strong objects.

Fig. 4 shows the monostatic image at $t_1 = 0.042$ ms. It exhibits a single peak. As usual, the data were inverted as a single object. That predicts a location $r_1 = (-28.07, -0.56, -54.82)$ cm which is not close to the location of either object. The resultant misfit is shown in row (a) of Fig. 4 along with the polarization curves. We note that there is considerable signal in the residual map, showing that the inversion was not successful in reproducing the data. Also, the recovered polarization curves do not match well those of any of the library items. This poor result suggests that the data might be better interpreted as being from two objects.

For a two-object inversion, it is understandable that a pair of locations in the 3-D Cartesian space is equivalent to one point in a 6-D model space. In the following, we will interchangeably use words "point" or "pair." First, we define the ROI that horizontally corresponds to the size of the sensor footprint and vertically extends from the surface or bottom of the sensor down to 1.36 m. We then randomly select 300 trial pairs of locations and evaluate their misfits Φ_d using (14). These points are sorted and plotted (circles) in Fig. 5(a). Fig. 5(b) shows Φ_d versus the depths z_1 and z_2 for the sampled points. We use these plots to select ten points that have a low misfit and also have distinct horizontal and vertical locations. These are shown by the crosses in Fig. 5(a) and (b). These ten selected trial locations serve as input to the nonlinear inversion to recover locations and polarization tensors. In this example, all selected trials converged to the same locations. As an illustration, Table I lists the first five initial and final locations ordered from the final Φ_d values that have subtle differences. Fig. 5(c) shows the convergence curve of Φ_d as a function of the iteration number starting from one (marked as No. 1 in Table I) of the trials, shown by a pair of circles in Fig. 5(d). It roughly took seven iterations to reduce the value of Φ_d from an initial value of 35 816 to 545. There were no significant changes in Φ_d with further iterations. In Fig. 5(d), the inverted locations, denoted as crosses $r_1 = (0.61, -0.74, -58.35)$ cm and $r_2 =$ $\left(-49.74,-0.52,-46.07\right)$ cm), are close to the true locations marked as diamonds.



Fig. 5. Example 1a of TEMTADS test-stand overlapping data: 4.2'' mortar + half shell with $H_d = 50$ cm. Determining source locations by the multistart algorithm. (a) (Circles, the vertical axis on the right) Ordered Φ_d values versus number of initial points; (crosses, the axis on the left) the top ten smaller Φ_d points. (b) Three-dimensional scatter plot of Φ_d versus depths z_1 and z_2 of the 300 (circles) initial pair points and the (crosses) selected. (c) Convergence curve of Φ_d as a function of the iteration number starting from one selected point. (d) Three-dimensional scatter plot of the (circles) selected starting locations, the (crosses) inverted locations, and the (diamond) true locations.

TABLE I EXAMPLE 1A OF TEMTADS TEST-STAND OVERLAPPING DATA: FIVE PAIRS OF INITIAL AND FINAL LOCATIONS. THE TRUE LOCATIONS: (0.0, 0.0, -61.0) cm and (-50.0, 0.0, -44.0) cm

No.	Label	(x_1, y_1, z_1) (cm)	(x_2, y_2, z_2) (cm)	Φ_d
1	Starting	(-9.47, 8.09, -60.17)	(-59.34, 31.55, -94.21)	35816.023
	Final	(0.61, -0.74, -58.35)	(-49.74, -0.52, -46.07)	545.716223
2	Starting	(-77.67, 15.28, -60.91)	(-31.08, -10.23, -69.14)	31659.93
	Final	(-49.74, -0.52, -46.07)	(0.61, -0.74, -58.35)	545.716224
3	Starting	(-18.16, -4.34, -59.42)	(-66.20, 48.37, -26.55)	21950.535824
	Final	(0.61, -0.74, -58.35)	(-49.74, -0.52, -46.07)	545.716224
4	Starting	(-19.36, 12.21, -55.26)	(38.73, -56.61, -124.22)	32679.154200
	Final	(-49.74, -0.52, -46.07)	(0.61, -0.74, -58.35)	545.716229
5	Starting	(50.26, 77.41, -74.68)	(-16.63, -15.01, -52.73)	40529.958228
	Final	(0.61, -0.74, -58.35)	(-49.74, -0.52, -46.07)	545.716238



Fig. 6. Examples 1 and 2 of TEMTADS test-stand overlapping data: 4.2'' mortar + nose piece. (In black and green solid curves) Inverted polarizabilities when (a) Example 1b of $H_d = 50$ cm, (b) Example 2a of $H_d = 10$ cm, and (c) Example 2b of $H_d = 0$ cm. Red and blue dash curves represent the known polarizations of 4.2'' mortar + nose piece shown in Fig. 3(a) and (c).

Having obtained the locations, the next step is to compute the orientation and principal polarizations for the objects. Using the methodology outlined earlier, the estimated Euler angles of the two objects are $(\phi_1, \theta_1, \psi_1) = (181.14^\circ, 88.77^\circ, 9.66^\circ)$ and $(\phi_2, \theta_2, \psi_2) = (182.97^\circ, 95.39^\circ, 0.05^\circ)$. This is an acceptable agreement with the ground truth where the objects were oriented horizontally.

In Fig. 4, row (b) presents the predicted data, residuals, and recovered polarizations after the two-object inversion. Visually, the multiobject inversion explains data well. Defining the relative error of data misfit as

$$\epsilon_d = \sqrt{\frac{\sum_{j=1}^N \left\| W_j \left(\mathbf{d}_{\text{obs}}(t_j) - \tilde{\mathbf{d}}(t_j) \right) \right\|^2}{\sum_{j=1}^N \left\| W_j \mathbf{d}_{\text{obs}}(t_j) \right\|^2}}$$
(21)

the single-object inversion has $\epsilon_d = 35.02\%$, while the twoobject inversion has $\epsilon_d = 6.53\%$. The two-object inversion has 81.35% fit improvement over the single-object inversion and accurately recovers the polarizations of each object as compared with the ground truth.

Other examples were done that also included the 4.2'' mortar + nose piece, and we found that when the separation was large, $H_d = 50$ cm, the multiobject inversion is able to accurately obtain the polarizations of strong and weak objects, shown as example 1b in Fig. 6(a). Similarly, at other large H_d separations, we can achieve satisfactory recovery of object polarizations for both 4.2'' mortar + half shell and 4.2'' mortar + nose piece configurations. The more challenging scenarios are to test if we can well recover the polarizations when two objects are much closer.



Fig. 7. Example 2c of TEMTADS test-stand overlapping data: 4.2'' mortar + half shell with $H_d = 10$ cm. Monostatic image at $t_1 = 0.042$ ms. The white dots in the images denote the center positions of the sensors. (a) Two-object inversion. (b) Single-object inversion. On the polarization plots, red and blue dash curves represent the known polarizations of 4.2'' mortar and half shell shown in Fig. 3(a) and (b).

2) Examples 2—Smaller Horizontal Separation: Here, we present the examples when two objects have a horizontal separation of $H_d = 10$ cm and of $H_d = 0$ cm.

We first consider the 4.2'' mortar + nose piece. The multiobject inversion produces accurate polarizations of 4.2'' mortar at these smaller separations (as examples 2a and 2b in Fig. 6(b) and (c)). However, for this small piece of clutter, its recovered polarizations at $H_d = 0$ cm becomes inaccurate.

We next consider the half shell at separations $H_d = 10$ cm and $H_d = 0$ cm. Notice that, in Fig. 3, the half shell has a comparable polarization strength with the 4.2" mortar. For the case of a 4.2" mortar and half shell at small separations, it is more difficult to get a good recovery of their polarizations.

For the case of $H_d = 10$ cm (example 2c), the two-object inversion in Fig. 7(a) achieves a smaller residual distribution than using a single-object model in Fig. 7(b) and predicts the two objects at $r_1 = (5.99, -1.18, -55.20)$ cm and $r_2 =$ (-14.06, -1.21, -48.63) cm. A single-object inversion predicts an object at $r_1 = (-6.51, -1.19 - 51.82)$ cm. Comparing with the true locations, $r_1^{\text{true}} = (0.00, 0.00, -61.0)$ cm and $r_{2}^{\text{true}} = (-10.00, 0.00, -44.0)$ cm, we see that all coordinates agree to within 6 cm and the polarizations of the objects are well recovered. In this case, likely because the polarizations of the two objects are quite similar, the single-object inversion produces an object that lies midway between the two items and the polarization curves are approximately those of the 4.2'' mortar. With only a single-object analysis, the regulators would have discovered the ordnance but would also have been surprised to find a second item.

When $H_d = 0$ cm (example 2d), the large piece of clutter is directly above the ordnance. The two-object inversion in Fig. 8(a) predicts the locations at $r_1 = (-10.07, -0.69, -51.71)$ cm and $r_2 = (7.64, -1.39, -51.20)$ cm, with

 $\epsilon_d = 5.84\%$. In this difficult case, the polarizations from the two-object inversion are inaccurately recovered. It predicts two identical rodlike objects in terms of polarization characteristics. The inversion tries to interpret the anomaly data as two objects at the same depth with horizontal separation of about 17 cm. In the single-object inversion, it predicts the location at $r_1 = (-0.73, -1.05, -51.92)$ cm with $\epsilon_d =$ 8.32% and recovers some features of the ordnance polarizations [Fig. 8(b)]. For understanding this overlapping problem, we conducted an inversion by manually specifying locations of the two objects at $r_1 = (0.61, -0.74, -58.35)$ cm and $r_2 =$ (-0.49, -0.52, -46.07) cm according to the ground truth and the inverted results of $H_d = 50$ cm. Fig. 8(c) shows that the polarizations of the two objects, particularly the polarizations for the ordnance, are much better recovered but the data fit is $\epsilon_d = 8.79\%$. Among the three inversion results, the automatic two-object inversion gives the smallest misfit but with inaccurate recovery of polarizations of the objects. Thus, the tests raise a question about how to select an inversion result from multiple solutions for certain difficult data anomalies. Judging solely upon a misfit-based metric may not always give the best solution. In such a case, examination of the recovered polarizabilities against the available library may provide sort of complementary information in making a decision about inversions.

3) Discussion Arising From Test-Stand Data: The numerous tests show that there are a number of factors that can affect our ability to resolve an overlapping anomaly. These factors include spatial separation of the underlying sources (e.g., deep and shallow, far and close) and their polarization strengths. In an attempt to understand our previous results, where we observed that the case of strong + weak configuration (e. g, mortar + nose piece) is better resolved than strong + strong configuration (e.g., mortar + half shell), we briefly



Fig. 8. Example 2d of TEMTADS test-stand overlapping data: 4.2'' mortar + half shell with $H_d = 0$ cm. Monostatic image at $t_1 = 0.042$ ms. The white dots in the images denote the center positions of the sensors. (a) Two-object inversion. (b) Single-object inversion. (c) Two-object inversion at the specified locations. On the polarization plots, red and blue dash curves represent the known polarizations of 4.2'' mortar and half shell shown in Fig. 3(a) and (b).

investigate the use of covariance of the recovered polarization tensor elements. We use the metric given by [45]

$$\epsilon_p = \operatorname{Trace}\left[(A^{\mathrm{T}}A)^{-1} \right] \tag{22}$$

where $A = [A(\mathbf{r}_1) \ A(\mathbf{r}_2)]$, as defined in (12). ϵ_p reflects the size of the total uncertainty of polarizabilities and, therefore, is a global measure of resolution. It is most informative when used in a relative sense to compare two sources. Equation (22) was similarly used in [13] and [46] to assess the relative merits of different transmitter/receiver combinations and to measure survey quality in EMI sensing. Here, we use (22) as a metric to characterize how two-object problems can be resolved by a specific array. For example, given two overlapping configurations, the one with a lower ϵ_p is likely better resolved by an array.

Using the TEMTADS array as a test, Fig. 9 shows ϵ_p as a function of horizontal separation H_d between two objects when their vertical separation is $V_d = 17$ cm and $V_d = 34$ cm. This emulates the test-stand configuration of Fig. 2. The ϵ_p values in the figure are normalized by ϵ_p at $V_d = 17$ cm and $H_d = 0$ cm. For the case of $V_d = 17$ cm, ϵ_p is reduced to 50% when $H_d = 10$ cm, and it is further reduced to around 15% for the large separation of $H_d = 50$ cm. Conservatively, for this smaller vertical separation, the minimum H_d that an



Fig. 9. Total uncertainty of polarizabilities against the horizontal separation between two objects. (a) $V_d = 17$ cm. (b) $V_d = 34$ cm.

overlapping anomaly can be necessarily resolved might be around $10 \sim 20$ cm. On the other hand, when the vertical separation between the two objects is increased to 34 cm, the normalized ϵ_p values become smaller at different H_d values and the corresponding curve is flatter. The largest normalized ϵ_p is only about 0.26 at $H_d = 0$. Probably, we would say that an overlapping anomaly with this large vertical separation, even with much smaller horizontal separation, can be resolved given an adequate signal-to-noise ratio. This seems consistent with the observations in our inversion tests in Fig. 6 for the case of 4.2'' mortar + nose piece.



Fig. 10. Monostatic observed data for anomaly 489 and the predicted data and residuals after the single-object inversion. The cross in one predicted image denotes the inverted location of an object. The white dots in the images denote the center positions of the sensors. The first row is for $t_1 = 0.042$ ms, and the second row is for $t_{53} = 0.99$ ms. On the polarization plot, red dash curves represent the polarizabilities of the 60-mm mortar in the library.

B. Field Data

Here, we present the applications of the multiobject inversion technique to field data that were acquired at San Luis Obispo (SLO), CA. In the field survey, the TEMTADS and MetalMapper sensor heights are at 0.3 and 0.21 m above the ground, respectively.

1) TEMTADS: The first example is to use a set of field data marked with ID 489, which was included in the training data. It was stated that the data are associated with a single object and the ground truth is a 60-mm mortar.

Fig. 10 shows, from left to the right, the observed, predicted, and residual data at (a) $t_1 = 0.042$ ms and (b) $t_{53} = 0.99$ ms using a single-object inversion. The recovered polarizations are also shown. There is a strong anomaly at the corner of the array that shows up in the early time channels, but it is not fit by the single-object model inversion. By 1 ms, we see mainly a centered anomaly with rather weak signal and the corner anomaly has nearly disappeared. It predicts an object near the center of the array at $r_1 = (0.06, -0.12, -0.50)$ m, denoted as a cross on the modeled data. However, the polarizations of the item in Fig. 10 are larger than those (in red) for a 60-mm mortar (the polarization curves for the 60 mm were obtained from inverting test-pit data), and they indicate a non-UXO object which has two approximately equal major polarizabilities. In summary, using a single-object model, we were unable to obtain a solution that has a sufficiently good fit to the data and we failed to get correct polarization signature of the target.

Next, we subjected the data to a two-object inversion. The results are shown in Fig. 11. Examining the predicted data and the residual distribution, we see that data are much better explained. The strong corner anomaly at early times and the center weak anomaly visible around 1 ms are well reproduced. With further inspection of the recovered polarizations, one would confidently infer that the two-object inversion pre-

dicts one axis-symmetric object at $r_1 = (0.12, 0.19, -0.32)$ m, whose polarizations (in black) match well with those of the 60-mm mortar (in red), and one clutter-like object at $r_2 =$ (0.81, -0.81, -0.06) m, whose polarizations (in green) show two almost equal major curves that are strong at early times but fade out quickly after around 1 ms. The polarization characteristics explain the signal of the anomaly near the corner.

Fig. 12 shows another example of applying the technique to data that could include two objects. The single- and twoobject inversions have the normalized misfit values of 0.849 and 0.932, which indicate that both equally fit well the observed data. The single-object predicts a shallow object at $r_1 =$ (-0.02, 0.02, -0.05) m, while the two-object inversion predicts that one shallow object at $r_1 = (-0.02, -0.02, -0.04)$ m and a deep object at $r_2 = (0.07, 0.01, -0.44)$ m. Their horizontal separation is roughly 9.5 cm. Based on the good match of the primary polarizability of the deep object with that in our library, we would infer that it is likely a 60-mm mortar. In fact, the inference agrees with the ground truth. From the single object inversion, the anomaly is most likely misclassified as a non-UXO.

2) MetalMapper: This is an example (with data ID of 1177) of misclassification that occurred using a single-object inversion. In a retrospect analysis, a two-object inversion was conducted to evaluate whether we can recover the polarizations of a discovered UXO. All of the MetalMapper data were inverted, and we have plotted a few of the 63 multicomponent measurements to show some data points where the difference between both inversion fittings is manifest under the same excitations, e.g., Tx-Z (i.e., the horizontal transmitter). In Fig. 13, one can see that the two-object inversion provides a far better data fit than the single-object inversion in data amplitudes and polarities, for example, see the components of RxZ-1, RxX-2, and RxY-2. Here, we use RxZ-1 to represent the



Fig. 11. Monostatic observed data for anomaly 489 and the predicted data and residuals after the two-object inversion. The two crosses in one predicted image denotes the inverted locations of two objects. The white dots in the images denote the center positions of the sensors. The first row is for $t_1 = 0.042$ ms, and the second row is for $t_{53} = 0.99$ ms. On the polarization plot, red dash curves represent the polarizabilities of the 60-mm mortar in the library.



Fig. 12. Monostatic observed data at $t_1 = 0.042$ ms for anomaly 1285 and the predicted data and residuals. The cross(es) in one predicted image denotes the inverted location(s) of an object or two objects. The white dots in the images denote the center positions of the sensors. (a) Single-object inversion. (b) Two-object inversion. On the polarization plot, red dash curves represent the polarizabilities of the 60-mm mortar in the library.

z-component of receiver cube 1. The same notation rule applies to other components. The single-object predicts the location of an object at (-0.16, 0.25, -0.19) m, while the two-object inversion predicts that the locations of the objects are at $r_1 =$ (-0.17, 0.21, -0.09) m and $r_2 = (-0.16, 0.63, -0.32)$ m. This corresponds to one shallow object and one deep object. In fact, the two-object inversion with the normalized misfit of 2.88 attains a significant data fit improvement of 79.5% over the single-object inversion that has a misfit of 14.07. Furthermore, by matching the recovered polarizations with our MetalMapper polarizability library, we can identify in Fig. 14 that the deep target is likely a 60-mm mortar. Again, this is an overlapping scenario where a small item of interest (here, a 60-mm mortar) is deeply buried and is overlain by two smaller 60-mm tail booms [33]. In this difficult case, the weaker signals arising from the deep 60-mm mortar were dominated by the responses of the two shallow 60-mm tail booms.

V. CONCLUSION

We have considered the problem of inverting multiple objects using TEM data. This is of practical interest in the cleanup of UXO-contaminated sites where items are closely spaced and can be sensed simultaneously within the field view of sensors.



Fig. 13. MetalMapper anomaly 1177. Some selected components from Tx-Z (the horizontal transmitter) excitation. (a) Single-object inversion. (b) Two-object inversion. Observation: (Blue cross) Positive; (red circle) negative. Prediction: (Green cross) Positive; (black circle) negative. In the top left subplot of (a) and (b), RxZ-1 represents the *z*-component of receiver cube 1. Similar notations appears in the other subplots.

Our methodology for this problem is to decompose a highdimensional model space into lower dimensional model spaces and solve smaller decomposed problems separately and sequentially. Numerically, the problem is cast into explicit separable nonlinear optimization problems in which a set of nonlinear model parameters, for example, the locations of the underlying sources, are sought while the set of linear model parameters, their polarizations, are updated accordingly in a nested manner.



Fig. 14. MetalMapper anomaly 1177. Recovered Polarizabilities. (a) Singleobject inversion. (b) Two-object inversion. Red curves represent polarizabilities in the library.

The technique, allowing to fix one set of model parameters and to optimize another set, can have the capability of mitigating the tradeoff between the different classes of model parameters. As part of the process, we have proposed a selected multistart nonlinear algorithm for source localizations that paves an efficient way to finding a good initial guess of the model parameters, helps guard against getting trapped in potential local minima, and makes the nonlinear inversion effectively automated.

The technique has been evaluated using data from the nextgeneration sensors, namely, the TEMTADS and MetalMapper systems. Using the TEMTADS test-stand data, we studied the spatial resolution of overlapping anomalies through inversions and with the metric defined as the total uncertainty of the polarizabilities. Spatial separation between two objects is a major factor to govern overlapping level of an anomaly. Generally, both vertical and horizontal separations are mutually dependent factors that influence the resolution of multiple objects. This initial study shows that an overlapping anomaly with a minimum 10 \sim 20 cm of horizontal separation might be resolved for a vertical separation of 17 cm. When the vertical separation of two objects is increased, even smaller horizontal separations may be resolved. We should mention that these detailed inferences are case dependent and that resolution numbers cannot be blindly used for other configurations and noise backgrounds. Rather, our work serves as a preliminary understanding of how we can resolve closely spaced objects.

The tests of the technique using the field data demonstrate its capability to recover target signatures of interest for a number of difficult overlapping anomalies (e.g., deep and shallow objects sensed together) and to be able to provide accurate input to classification analysis for improving discrimination. Thus, the technique can become a practical tool in UXO clearance projects.

ACKNOWLEDGMENT

The authors would like to thank D. Snyder of Snyder Geoscience, Inc., and J. Kingdon of SAIC for the valuable help and discussions in the data processing with the new sensor systems, D. Steinhurst of Nova Research, Inc., for providing us the highquality test-stand data for the evaluation of our technique, our colleagues L. Beran, N. Lhomme, K. Kingdon, and D. Sinex for the great help and work in the processing of the SLO field data, and the editor and reviewers for their constructive comments that help improve the paper.

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