Cooperative Magnetic Inversion

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Outline

- Review on magnetics
- Effect of remanence
- Review on algorithms
  - Amplitude Inversion
  - Magnetic Vector Inversion
- Cooperative strategy
- Ongoing research
Magnetic Data

- Magnetization

\[ \vec{M} = \kappa (\vec{H}_0 + \vec{H}_s) + \vec{M}_{nrm} \]

- \( \kappa \vec{H}_0 \): induced
- \( \kappa \vec{H}_s \): self – demag
- \( \vec{M}_{NRM} \): Remanence
Magnetic Data

- Magnetic Data

\[
\vec{b}(P) = \frac{\mu_0}{4\pi} \int_V \vec{M} \cdot \nabla \nabla \left( \frac{1}{r} \right) dV
\]

Forward Simulation

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Magnetic Inversion

- Inverse Problem

This talk

$$\min_m \left\| F[m] - d^{obs} \right\|_2^2 + \beta \phi_m$$

Data misfit + Regularization

Next talk!! (4:10 pm)
Magnetic Susceptibility Inversion

- Induced assumption: (Li & Oldenburg 1996)

\[ \mathbf{M} = \mathbf{H}_0 \kappa \]

\[ \mathbf{b} = \mathbf{T} \mathbf{H}_0 \kappa \]
Magnetic Susceptibility Inversion

- Induced assumption

\[ \mathbf{M} = \mathbf{H}_0 \kappa \]

\[ \mathbf{b} = \mathbf{T} \mathbf{H}_0 \kappa \]

- Stable and robust for any inducing field orientation
- Several commercial (MAG3D) and open-source codes available
- Extensively used by the mineral exploration sector.
Magnetic Susceptibility Inversion

\[ \mathbf{M} = H_0 \kappa + \mathbf{M}_{nrm} \]

\[ \mathbf{b} = T H_0 \kappa \]
Magnetic Susceptibility Inversion

• Complication with remanence
  
  – Wrong assumption results in severe distortion
  
  – Common for many types of mineral deposits (Kimberlite, BIF, VMS)

\[ \mathbf{M} = \mathbf{H}_0 \kappa + \mathbf{M}_{nrm} \]

\[ \mathbf{b} = \mathbf{T} \mathbf{H}_0 \kappa \]
Previous Work

• Data processing
  – Fourier domain (Roest et al. 1992)
  – Helbig’s method (Phillips 2003)
  – Cross-correlation (Dannmiller & Li 2006)

• Simple and fast
• Becomes hard for multiple anomalies and requires gridding
Previous Work

• Amplitude Inversion
  - Invert data weakly sensitive to magnetization
    \[ |\vec{b}| = \left( b_x^2 + b_y^2 + b_z^2 \right)^{1/2} \]
    \[ \overline{\mathbf{M}} \approx \overline{\mathbf{H}}_0 \kappa_e \]
  - Major work from CSM
    Sarah Shearer, MSc. Thesis (2005)
    Li et al. 2010
    Li & Li 2014
Previous Work

- Amplitude Inversion: Li et al. 2014

\[ \overrightarrow{M} \approx \overrightarrow{H}_0 \kappa_e \]

\[ |\overrightarrow{b}| = \mathbf{F}[m] \kappa_e \]

Sensitivity weights applied

\[ W_r = \left[ \sum_{i=1}^{nd} J[i]^2 \right]^{1/2} \]
Previous Work

• Amplitude Inversion

\[ |\vec{b}| = \left[ b_x^2 + b_y^2 + b_z^2 \right]^{1/2} \]

– Robust recovery of effective susceptibility
– Requires pre-processing for amplitude data
Previous Work

- Magnetic Vector Inversion, Cartesian (MVI-C)
  - Directly for the components of magnetization
  
  \[
  \vec{M}_{pst} = \begin{bmatrix} H_p & H_s & H_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix}
  \]

- Lelievre & Oldenburg (2009)
- Geosoft VOXI
Magnetic Inversion

- Magnetization Vector Inversion, Cartesian (MVI-C)

\[
\vec{b} = T \vec{M}_{pst}
\]

\[
\varphi_m = \varphi_p + \varphi_s + \varphi_t
\]

\[
= \lVert p - p_{ref} \rVert_2 + \lVert \nabla p \rVert_2
+ \lVert s - s_{ref} \rVert_2 + \lVert \nabla s \rVert_2
+ \lVert t - t_{ref} \rVert_2 + \lVert \nabla t \rVert_2
\]
Previous Work

- Magnetic Vector Inversion, Cartesian (MVI-C)

\[ \mathbf{M}_{pst} = \begin{bmatrix} \mathbf{H}_p & \mathbf{H}_s & \mathbf{H}_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix} \]

- Recovers both amplitude and orientation
- Largely underdetermined, relies heavily on regularization
- Models too smooth for easy interpretation
- Clustering methods (FMC) (Li & Sun 2016)
Parallel research

- Compact Norms (Next talk @ 4:10 pm)

\[ \mathbf{M} = \mathbf{H}_0 \kappa \]

\[ \| \mathbf{f}(m) \|_p^p = \sum_{i=1}^{nc} |\mathbf{f}(m_i)|^p \]
Parallel research

• Limitations with current formulation
  – Sparsity imposed on individual components favoring magnetization along single component:

\[ \vec{M}_{pst} = \begin{bmatrix} H_p & H_s & H_t \end{bmatrix} \begin{bmatrix} \kappa_p \\ \kappa_s \\ \kappa_t \end{bmatrix} \]

  – Difficult to incorporate borehole constraints (koenigsberger ratio)
Cooperative Strategy

- Cooperative Magnetic Inversion (CMI)
  
  Fournier, MSc. 2015
  Liu et al. 2015

1. Susceptibility Inversion
2. Amplitude Inversion
3. MVI

Equivalent Source
Effective Susceptibility

Forward Simulation

Effective Susceptibility

EW Section
Cooperative Strategy

- Cooperative Magnetic Inversion (CMI)

Weighted regularization

\[ \varphi_m = \|W_m W_s (m - m_{\text{ref}})\|_2^2 + \sum_{i=x,y,z} \|W_m W_i G_i m\|_2^2 \]

Model weight

\[ W_m = \left[ \frac{\kappa_e}{\max(\kappa_e)} + \varepsilon \right]^{-1} \]
Cooperative Strategy

• Cooperative Magnetic Inversion (CMI)
  – Able to apply sparsity
  – Recover both the amplitude and direction
  – Minimum of 3 inversions required
  – Dependents on the ability of extracting amplitude data
  – Can’t apply sparsity on the orientation
Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Separate the amplitude and direction

\[ \hat{x} = a \cos \varphi \cos \theta \]
\[ \hat{y} = a \cos \varphi \sin \theta \]
\[ \hat{z} = a \sin \varphi \]

- Lelievre’s PhD. (2009)
- Put aside due to high non-linearity
Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Natural separate the amplitude and direction
  \[
  \begin{align*}
  \hat{x} &= a \cos \varphi \cos \theta \\
  \hat{y} &= a \cos \varphi \sin \theta \\
  \hat{z} &= a \sin \varphi
  \end{align*}
  \]
  - Solved issues regarding the non-linearity

  \[
  \varphi_m = \varphi_a + \varphi_t + \varphi_p = \|a - a_{\text{ref}}\|_2 + \|v_a\|_2 \\
  + \|t - t_{\text{ref}}\|_2 + \|v_t\|_2 \\
  + \|p - p_{\text{ref}}\|_2 + \|v_p\|_2
  \]

  Sensitivity:
  \[
  J = \frac{\partial d}{\partial m} = SG
  \]
  \[
  S = \begin{bmatrix}
  \cos \varphi \cos \theta & -a \sin \varphi \cos \theta & -a \cos \varphi \sin \theta \\
  \cos \varphi \sin \theta & -a \sin \varphi \sin \theta & a \cos \varphi \cos \theta \\
  \sin \varphi & -a \sin \varphi \sin \theta & \cos \varphi \\
  \end{bmatrix}
  \]
Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Apply Koenigsberger ratio constraints

\[ \varphi_m = \|a - a_{\text{ref}}\|_2 + \|\nabla a\|_2 + \|\nabla t\|_2 + \|\nabla p\|_2 \]
Ongoing research

• Magnetic Vector Inversion, Spherical (MVI-S)
  – Sparsity on amplitude

\[ \varphi_m = \| a - a_{\text{ref}} \|_p + \| \nabla a \|_p + \| \nabla t \|_2 + \| \nabla p \|_2 \]
Ongoing research

- Magnetic Vector Inversion, Spherical (MVI-S)
  - Sparsity on angles

\[ \varphi_m = \|a - a_{\text{ref}}\|_p + \|\nabla a\|_p + \|\nabla t\|_p + \|\nabla p\|_p \]
Case History

• Osborne Cu-Au deposit, Queensland
  – Airborne mag discovery in 1974 (Newmont Pty Ltd)
  – Magnetite-rich Ironstones layers within metasediments of Mt Isa quartzite.
Case History

• Strong self-demagnetization effects as studied by Logan & Anderson (1992)
Case History

- Conventional Inversions
Case History

- Conventional Inversions
Summary

- Managed to reduce the non-uniqueness of usual MVI-Cartesian formulation
- Allow to apply sparse norms
- Promising results with MVI-Spherical formulation
- Requires more work to improve performance
Thank you!