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3-D inversion of magnetic data

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ABSTRACT

We present a method for inverting surface magnetic data to recover 3-D susceptibility models. To allow the maximum flexibility for the model to represent geologically realistic structures, we discretize the 3-D model region into a set of rectangular cells, each having a constant susceptibility. The number of cells is generally far greater than the number of the data available, and thus we solve an underdetermined problem. Solutions are obtained by minimizing a global objective function composed of the model objective function and data misfit. The algorithm can incorporate a priori information into the model objective function by using one or more appropriate weighting functions. The model for inversion can be either susceptibility or its logarithm. If susceptibility is chosen, a positivity constraint is imposed to reduce the nonuniqueness and to maintain physical realizability. Our algorithm assumes that there is no remanent magnetization and that the magnetic data are produced by induced magnetization only. All minimizations are carried out with a subspace approach where only a small number of search vectors is used at each iteration. This obviates the need to solve a large system of equations directly, and hence earth models with many cells can be solved on a deskside workstation. The algorithm is tested on synthetic examples and on a field data set.

INTRODUCTION

Magnetic surveying has been used widely over the years, resulting in a great amount of data with enormous areal coverage. Magnetic data have been used for mapping geological structures, especially in the reconnaissance stage of exploration, but when used in detailed prospecting, robust and efficient inversion algorithms must be used. However, a principal difficulty with the inversion of the potential data is the inherent nonuniqueness. By Gauss' theorem, if the field distribution is known only on a bounding surface, there are infinitely many equivalent source distributions inside the boundary that can produce the known field. Any magnetic field measured on the surface of the earth can be reproduced by an infinitesimally thin zone of magnetic dipoles beneath the surface. From a mathematical perspective, this means there is no depth resolution inherent in magnetic field data. A second source for nonuniqueness is the fact that magnetic observations are finite in number and are inaccurate. If there exists one model that reproduces the data, there are other models that will reproduce the data to the same degree of accuracy. The severity of the nonuniqueness problem for magnetic data is illustrated in Figures 1–3. (The gray scale in all figures indicates susceptibility in SI units for model sections and magnetic data in nT for data plots.) A 3-D dipping prism of uniform susceptibility in Figure 1 produces the surface magnetic field shown in Figure 2, which consists of 441 data. Slices of a 3-D susceptibility model that adequately reproduces the 441 data are shown in Figure 3. That result, however, bears little resemblance to the true model. Susceptibility is concentrated near the surface and displays zones of negative values. This mathematical model solution provides little information about the true structure that is useful.

Faced with this extreme nonuniqueness, previous authors have mainly taken two approaches in the inversion of magnetic data. The first is parametric inversion, where the parameters of a few geometrically simple bodies are sought in a nonlinear inversion and values are found by solving an overdetermined problem. This methodology is suited for anomalies known to be generated by simple causative bodies, but it requires a great deal of a priori knowledge about the source expressed in the form of an initial parameterization, an initial guess for parameter values, and limits on the susceptibility allowed (e.g., Bhattacharyya, 1980; Zeyen and Pous, 1991). Nonuniqueness is not generally an issue because only a small subset of possible models is considered due to the restrictive nature of the inversion algorithm. A related, but unique, approach in Wang and Hansen (1990) assumes polyhedronal causative bodies and

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inverts for the position of the vertices of these bodies using the spectrum of the magnetic data. The method is general in principle but has difficulties both in constructing the causative bodies from the recovered vertices and in obtaining the susceptibility distribution.

In the second approach to inverting magnetic data, the earth is divided into a large number of cells of fixed size but of unknown susceptibility. Nonuniqueness of solution is recognized and the algorithm produces a single model by minimizing an objective function of the model subject to fitting the data. Green (1975) minimizes a weighted model norm with respect to a reference model, and this allows the interpreter to guide the inversion by varying the weighting according to the available information. Last and Kubik (1983) choose to minimize the total volume of the causative body so that the final model is compact and structurally simple. Guillen and Menichetti (1984) minimize the moment of inertia of the causative body with respect to the center of gravity or an axis passing through it. Their inversion result is guided by the estimate of the central depth and dip of the causative body. These approaches have merit but they are not flexible enough to handle problems we are concerned with. This is especially true of methods that attempt to collapse the anomalous susceptibility into a single body; such a solution is rarely an adequate representation of geologic structure.

In our inversion approach, we first make a decision about the variable in which the interpretation is to be made, that is, whether susceptibility, log susceptibility, or some function of susceptibility is sought. Next, we form a multicomponent objective function that has the flexibility to generate different types of models. The form of this objective function is such that it can correct for the undesirable aspects of the mathematically acceptable model in Figure 3, namely—the concentration of susceptibility near the surface, the excessive structure, and the existence of negative susceptibilities. Our objective function incorporates an optional reference model so that the constructed model is close to that. It penalizes roughness in three spatial directions, and it has a depth weighting designed to distribute the susceptibility with depth. Additional 3-D weighting functions in the objective function can be used to incorporate further information about the model. Such information might be available from other geophysical surveys, geological data, or the interpreter's qualitative or quantitative understanding of the geologic structure and its relation to the magnetic susceptibility. These 3-D weighting functions can also be used to answer questions about the existence of susceptibility features found from previous inversions. Negative susceptibilities are prevented by making a transformation of

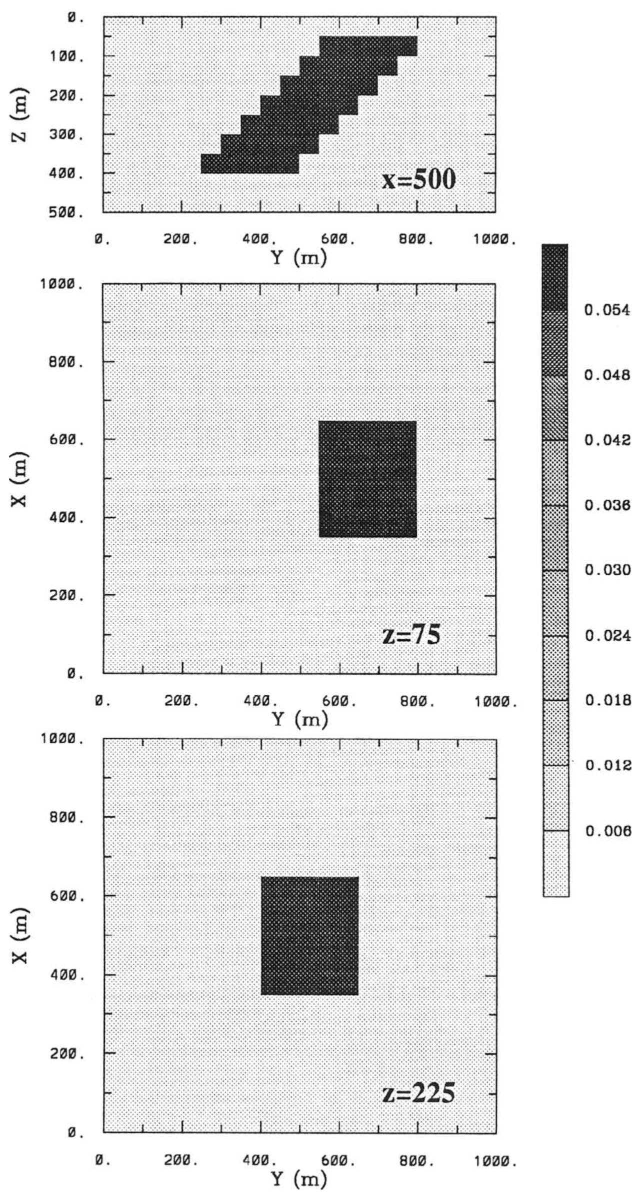

FIG. 1. Slices through a 3-D magnetic susceptibility model composed of a dipping slab in a nonsusceptible half-space. The slab is buried at a depth of 50 m and extends to 400-m depth at a dip angle of 45°. The gray scale indicates the value of magnetic susceptibility in SI units.

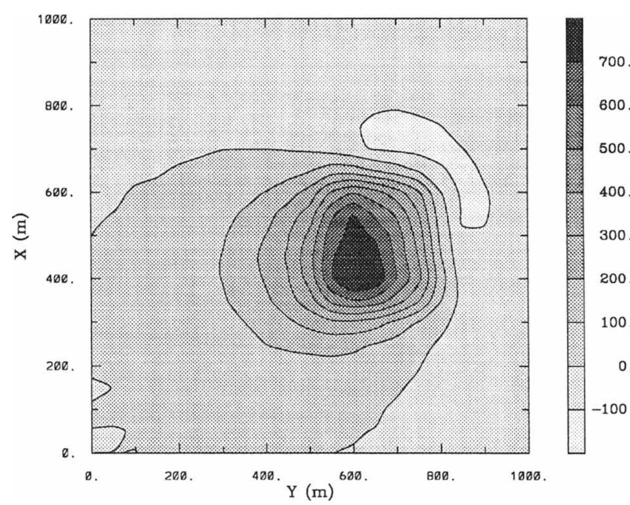

FIG. 2. The total field anomaly produced by the slab model in Figure 1. The inducing field has direction $I = 75°$ and $D = 25°$ and a strength of 50 000 nT. Uncorrelated Gaussian noise, with a standard deviation of 2% of the datum magnitude plus 1 nT, is added to the data. The gray scale indicates the magnetic anomaly in nT.

variables and solving a nonlinear inverse problem. The numerical solution for the inversion is accomplished by dividing the earth into a large number of cells so that relatively complex geologic bodies can be constructed. The computational difficulties often encountered in solving large matrix systems are avoided by working explicitly with a generalized subspace algorithm.

The paper begins by outlining our inversion methodology and empirically estimating parameters for the depth weighting based upon synthetic inversion of single 3-D prisms. Data from two synthetic models are then inverted. The paper concludes by inverting a field data set over a copper-gold porphyry deposit and a subsequent discussion.

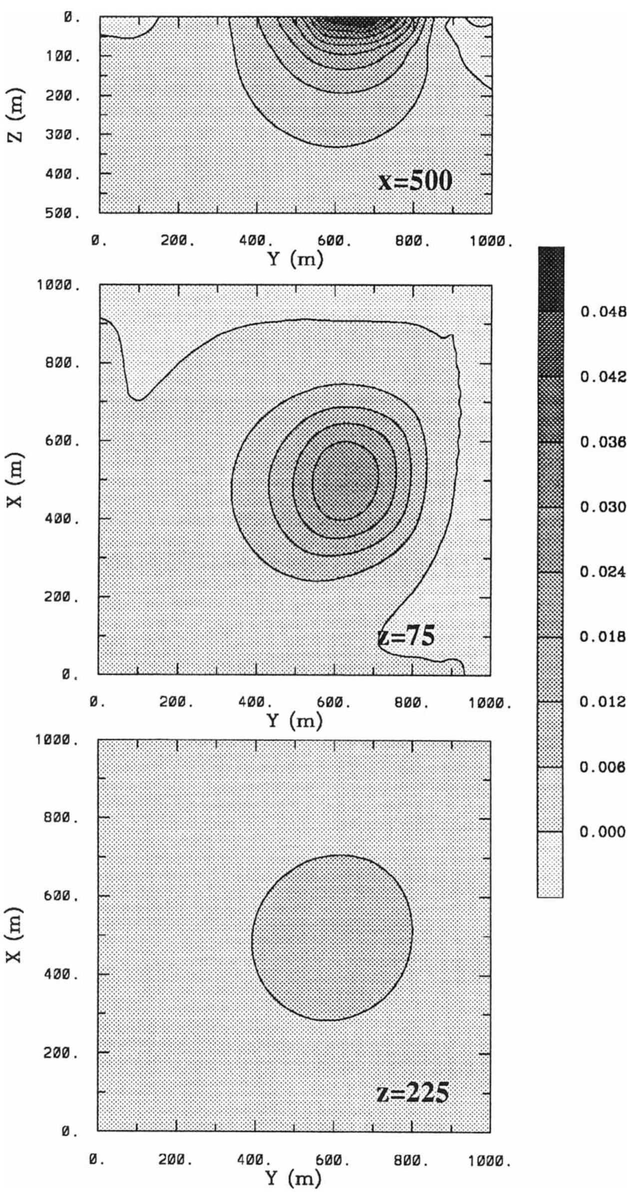

FIG. 3. The susceptibility model constructed by minimizing $\|\boldsymbol{\kappa}\|^2$ subject to fitting the data in Figure 2. As a mathematical solution, this model provides little, if any, information about the subsurface susceptibility distribution. It effectively illustrates the nonuniqueness inherent to the inversion of static magnetic field data.

INVERSION METHODOLOGY

Each magnetic anomaly datum observed above the surface can be evaluated by calculating the projection of the anomalous magnetic field onto a given direction. Let the source region be divided into a set of rectangular cells by an orthogonal 3-D mesh and assume a constant magnetic susceptibility value κ within each cell. Further we assume that there is no remanent magnetization and that the demagnetization effect is negligible. Thus only the induced magnetization is considered. This magnetization is uniform within each cell and is given by the product of the susceptibility and the inducing geomagnetic field $\underset{\sim}{\mathbf{H}}$. The magnetic anomaly at a location on, or above, the surface is related to the subsurface susceptibility by a linear relationship

$$\mathbf{d} = \underset{\sim}{\mathbf{G}}\boldsymbol{\kappa}, \tag{1}$$

where $\mathbf{d} = (d_1, \ldots, d_N)^T$ is the data vector and $\boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_M)^T$ is the susceptibility in the cells. The matrix $\underset{\sim}{\mathbf{G}}$ has as elements g_{ij}, which quantify the contribution of a unit susceptibility in the jth cell to the ith datum. Closed form solutions for g_{ij} were first presented in Bhattacharyya (1964) and later simplified in Rao and Babu (1991) into a form more suitable for fast computer implementation. The function g_{ij} is the projection onto a given direction of the magnetic field that is produced by a rectangular cell, so equation (1) is valid for computing different magnetic anomalies. For example, a projection onto the vertical direction gives the vertical magnetic anomaly while a projection onto the ambient geomagnetic field direction yields the total magnetic anomaly. Thus, the method presented here can be used to invert different types of magnetic data and in the following, we simply refer to them as the magnetic data with the understanding that it is direction specific.

Our inverse problem is formulated as an optimization problem where an objective function of the model is minimized subject to the constraints in equation (1). For magnetic inversion, the first question that arises concerns definition of the "model." Two possible choices are κ and ln (κ), but any function $g(\kappa)$ can, in principle, be used. In general, we prefer to invert for κ since the field anomaly is directly proportional to the susceptibility that varies on a linear scale. But depending upon the expected dynamic range of susceptibility and the physical interpretation attached to its value or variation, it may be that ln (κ) is more desirable. To accommodate this, we introduce the generic symbol m for the model with the understanding that it might be κ, ln (κ), or any monotonic function $g(\kappa)$. Having defined a model, we next construct an objective function, which when minimized, produces a model that is geophysically interpretable. The details of the objective function are problem dependent, but generally we need the flexibility to be close to a reference model m_0 and also require that the model be relatively smooth in three spatial directions. Here we adopt a right-handed Cartesian coordinate system with x positive north and z positive down. Let the model objective function be

$$\phi_m(m) = \alpha_s \int_V w_s\{w(z)[m(\mathbf{r}) - m_0]\}^2\, dv$$
$$+ \alpha_x \int_V w_x\left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial x}\right\}^2 dv$$
$$+ \alpha_y \int_V w_y\left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial y}\right\}^2 dv$$
$$+ \alpha_z \int_V w_z\left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial z}\right\}^2 dv, \qquad (2)$$

where functions w_s, w_x, w_y, and w_z are spatially dependent weighting functions while α_s, α_x, α_y, and α_z are coefficients that affect the relative importance of different components in the objective function. Here, $w(z)$ is a depth weighting function. It is convenient to write equation (2) as $\phi_m(m) = \phi_{ms} + \phi_{mv}$, where ϕ_{ms} refers to the first term in equation (2), and ϕ_{mv} refers collectively to the remaining three terms that involve variation of the model in three spatial directions.

The objective function in equation (2) has the flexibility of constructing many different models. The reference model m_0 may be a general background model that is estimated from previous investigations, or it could be the zero model. The reference model would generally be included in ϕ_{ms} but can be removed if desired from any of the remaining terms. Often we are more confident in specifying the value of the model at a particular point than in supplying an estimate of the gradient. The relative closeness of the final model to the reference model at any location is controlled by the function w_s. For example, if the interpreter has high confidence in the reference model at a particular region, he or she can specify w_s to have increased amplitude there compared to other regions of the model. The weighting functions w_x, w_y, and w_z can be designed to enhance or attenuate structures in various regions in the model domain. If geology suggests a rapid transition zone in the model, then a decreased penalty for variation can be put there, and the constructed model will exhibit higher gradients provided that this feature does not contradict the data. Therefore, the reference model and four 3-D weighting functions allow for the incorporation into the inversion of additional information other than the magnetic data. The additional information can be from previous knowledge about the susceptibility, from other geophysical surveys, or from the interpreter's qualitative or quantitative understanding about the geologic structure and its relation to susceptibility. When this extra information is incorporated, the inversion derives a model that not only fits the data, but more importantly, also has a likelihood of representing the earth. From the viewpoint of magnetic inversion, such an approach allows one to construct a most-likely earth model that uses all available information, and it can also be used to explore the nonuniqueness. These two aspects form the foundation of a responsible interpretation.

The kernels (values of g_{ij}) for the surface magnetic data decay with depth. It is for this reason that an inversion that minimizes $\|m - m_0\|^2 = \int (m - m_0)^2\, dv$ subject to fitting the data will generate a susceptibility that is concentrated near the surface. To counteract the geometric decay of the kernels and to distribute susceptibility with depth, we introduce a weighting of the form $w(z) = (z + z_0)^{-\beta/2}$ into ϕ_{ms}, and optionally include it in ϕ_{mv}. The values of β and z_0 are investigated in the following section, but their choice essentially allows equal chance for cells at different depths to be nonzero.

The next step in setting up the inversion is to define a misfit measure. Here we use the 2-norm measure

$$\phi_d = \|\underset{\sim}{\mathbf{W}}_d(\mathbf{d} - \mathbf{d}^{obs})\|_2^2, \qquad (3)$$

and we assume that the contaminating noise on the data is independent and Gaussian with zero mean. Specifying $\underset{\sim}{\mathbf{W}}_d$ to be a diagonal matrix whose ith element is $1/\sigma_i$ where σ_i is the standard deviation of the ith datum, makes ϕ_d a chi-squared variable distributed with N degrees of freedom. Accordingly $E[\chi^2] = N$ provides a target misfit for the inversion.

The inverse problem is solved by finding a model m that minimizes ϕ_m and misfits the data by a predetermined amount. This is accomplished by minimizing $\phi(m) = \phi_m + \lambda^{-1}(\phi_d - \phi_d^*)$, where ϕ_d^* is our target misfit and λ is a Lagrangian multiplier. To perform a numerical solution, we first discretize the objective function in equation (2) using a finite-difference approximation according to the mesh defining the susceptibility model. This yields

$$\phi_m(\mathbf{m}) = \phi_{ms} + \phi_{mv} = (\mathbf{m} - \mathbf{m}_0)^T\underset{\sim}{\mathbf{W}}_s^T\underset{\sim}{\mathbf{W}}_s(\mathbf{m} - \mathbf{m}_0) + (\mathbf{m} - \mathbf{m}_0)^T(\underset{\sim}{\mathbf{W}}_x^T\underset{\sim}{\mathbf{W}}_x + \underset{\sim}{\mathbf{W}}_y^T\underset{\sim}{\mathbf{W}}_y + \underset{\sim}{\mathbf{W}}_z^T\underset{\sim}{\mathbf{W}}_z)(\mathbf{m} - \mathbf{m}_0)$$
$$\equiv (\mathbf{m} - \mathbf{m}_0)^T\underset{\sim}{\mathbf{W}}_m^T\underset{\sim}{\mathbf{W}}_m(\mathbf{m} - \mathbf{m}_0) = \|\underset{\sim}{\mathbf{W}}_m(\mathbf{m} - \mathbf{m}_0)\|^2, \qquad (4)$$

where $\mathbf{m}$ and $\mathbf{m}_0$ are M-length vectors. The individual matrices $\underset{\sim}{\mathbf{W}}_s$, $\underset{\sim}{\mathbf{W}}_x$, $\underset{\sim}{\mathbf{W}}_y$, $\underset{\sim}{\mathbf{W}}_z$ are calculated straightforwardly once the model mesh and the weighting functions w_s, w_x, w_y, w_z, and $w(z)$ are defined (see Appendix). The cumulative matrix $\underset{\sim}{\mathbf{W}}_m^T\underset{\sim}{\mathbf{W}}_m$ is then formed. For our formulation, the matrix $\underset{\sim}{\mathbf{W}}_m$ is never computed explicitly but we shall use it to derive our final equations.

The inverse problem is solved by minimizing $\phi(\mathbf{m})$ with an appropriate minimization technique. To reduce computation and to invoke positivity, we use a subspace methodology. In its general form, the subspace technique allows the model parameter to be both positive and negative, and thus to ensure positive susceptibility, we may need to invoke a transformation of variables. Whether or not the transformation is required depends upon the relationship between m_i and κ_i. If $m_i = \ln(\kappa_i)$, so that interpretations are carried out in the logarithmic domain, then no further transformation is necessary since κ_i will be positive irrespective of the sign of m_i. However, if $m_i = \kappa_i$, or $m_i = g(\kappa_i)$, and $g(\kappa)$ is a positive function, then a transformation is required. All possibilities can be handled by introducing a new parameter p, such that $m_i = f(p_i)$, where $f(p)$ is a monotonic function whose inverse and first-order derivative exist. This mapping is then incorporated directly into the subspace minimization process.

Let $\mathbf{p}^{(n)}$ denote the parameter vector at the nth iteration and δp denote the sought perturbation. Performing a Taylor expansion of the perturbed model objective function about the point $\mathbf{p}^{(n)}$ yields

$$\phi_m(\mathbf{p}^{(n)} + \delta p) = \|\underset{\sim}{\mathbf{W}}_m \underset{\sim}{\mathbf{F}} \delta p + \underset{\sim}{\mathbf{W}}_m(\mathbf{m}^{(n)} - \mathbf{m}_0)\|^2, \tag{5}$$

where $\underset{\sim}{\mathbf{F}}$ is a diagonal matrix with elements

$$F_{ii} = \left.\frac{\partial f_i}{\partial p_i}\right|_{p^{(n)}} = \left.\frac{\partial m_i}{\partial p_i}\right|_{p^{(n)}}. \tag{6}$$

A similar Taylor expansion applied to the misfit objective functional $\phi_d(\mathbf{p}^{(n)} + \delta p)$ yields

$$\phi_d = \|\underset{\sim}{\mathbf{W}}_d \underset{\sim}{\mathbf{G}} \underset{\sim}{\mathbf{F}} \delta p + \underset{\sim}{\mathbf{W}}_d(\mathbf{d}(\mathbf{p}^{(n)}) - \mathbf{d}^{obs})\|^2. \tag{7}$$

At each iteration we desire a perturbation that minimizes equation (4) subject to generating a data misfit of $\phi_d = \phi_d^{*(n)}$, where $\phi_d^{*(n)}$ is the target misfit at the nth iteration. In the subspace technique we represent the perturbation as

$$\delta p = \sum_{i=1}^{q} \alpha_i \mathbf{v}_i \equiv \underset{\sim}{\mathbf{V}} \boldsymbol{\alpha}, \tag{8}$$

where the M-length vectors $\mathbf{v}_i (i = 1, q)$ are as yet arbitrary. Writing the objective function to be minimized in terms of the coefficients $\boldsymbol{\alpha}$ yields

$$\phi(\boldsymbol{\alpha}) = \|\underset{\sim}{\mathbf{W}}_m \underset{\sim}{\mathbf{F}} \underset{\sim}{\mathbf{V}} \boldsymbol{\alpha} + \underset{\sim}{\mathbf{W}}_m(\mathbf{m}^{(n)} - \mathbf{m}_0)\|^2 + \lambda^{-1}(\|\underset{\sim}{\mathbf{W}}_d \underset{\sim}{\mathbf{G}} \underset{\sim}{\mathbf{F}} \underset{\sim}{\mathbf{V}} \boldsymbol{\alpha} + \underset{\sim}{\mathbf{W}}_d(\mathbf{d}(\mathbf{p}^{(n)}) - \mathbf{d}^{obs})\|^2 - \phi_d^*). \tag{9}$$

Differentiating with respect to the coefficients $\boldsymbol{\alpha}$ yields the final equations

$$\begin{aligned} \underset{\sim}{\mathbf{B}} \boldsymbol{\alpha} &= \mathbf{b}, \\ \underset{\sim}{\mathbf{B}} &= \underset{\sim}{\mathbf{V}}^T \underset{\sim}{\mathbf{F}}^T (\underset{\sim}{\mathbf{G}}^T \underset{\sim}{\mathbf{W}}_d^T \underset{\sim}{\mathbf{W}}_d \underset{\sim}{\mathbf{G}} + \lambda \underset{\sim}{\mathbf{W}}_m^T \underset{\sim}{\mathbf{W}}_m) \underset{\sim}{\mathbf{F}} \underset{\sim}{\mathbf{V}}, \\ \mathbf{b} &= -\underset{\sim}{\mathbf{V}}^T \underset{\sim}{\mathbf{F}}^T \underset{\sim}{\mathbf{G}}^T \underset{\sim}{\mathbf{W}}_d^T \underset{\sim}{\mathbf{W}}_d (\mathbf{d}^{(n)} - \mathbf{d}^{obs}) \\ &\quad - \lambda \underset{\sim}{\mathbf{V}}^T \underset{\sim}{\mathbf{F}}^T \underset{\sim}{\mathbf{W}}_m^T \underset{\sim}{\mathbf{W}}_m (\mathbf{m}^{(n)} - \mathbf{m}_0). \end{aligned} \tag{10}$$

We note that the matrix $\underset{\sim}{\mathbf{B}}$ is $q \times q$ and therefore the system of equations is easily solved if q is small. At each iteration, we search for a value of λ that yields the target misfit for that iteration. If the target misfit cannot be reached, then the value of λ that achieves the smallest misfit is taken. The search is usually accomplished by solving equation (10) a number of times using different λ values. Once the optimum value of λ is found, the system is solved again to obtain the coefficients $\boldsymbol{\alpha}$ and the model perturbation. This iterative process is continued until the final expected data misfit is achieved and the model objective function undergoes no significant decrease with successive iterations. Subspace vectors $\mathbf{v}_i$ are generated mainly from the gradients of the data and model objective functions. The data are grouped to form subobjective functions of misfit, and a steepest descent vector corresponding to each subobjective function is used as a subspace vector. Partitioning of the data can be formed by grouping data that are spatially close, or by grouping data such that each group has approximately the same contribution to the total data misfit. Both approaches have worked well. The model objective function is partitioned and the gradient vector associated with each of the four components in the model objective function provides additional subspace vectors. In addition, a constant vector is always included, and the selected subspace vectors are orthonormalized before being used in the search. More details on the implementation of the subspace method for the linear inverse problem can be found in Oldenburg and Li (1994).

The final item of practical importance is the specification of the mapping needed to ensure positivity of susceptibility. The positivity is required since we are dealing only with induced magnetization, and the presence of negative susceptibility is negligible in practical geophysical applications. Although our formalism permits the minimization of $m = g(\kappa)$, the two most common situations are $m = \ln(\kappa)$ and $m = \kappa$. When $m = \ln(\kappa)$, we set $p = m$ and hence the matrix $\underset{\sim}{\mathbf{F}}$ in equation (10) is the identity matrix. If $m = \kappa$, we use the two-stage mapping proposed in Oldenburg and Li (1994). It is composed of an exponential segment and a straight line. The two segments are joined together such that the mapping and its first derivative are both continuous. The mapping is given by

$$\kappa = \begin{cases} 0 & p < p_b \\ e^p - \kappa_b & p_b \le p \le p_1, \\ (p - p_1 + 1)e^{p_1} - \kappa_b & p > p_1 \end{cases} \tag{11}$$

where $p = p_1$ is the transition point between exponential and linear segments, and κ_b is selected to be small enough such that susceptibilities smaller than κ_b are not significantly different from zero when the final interpretation is carried out. Here, κ_1 and hence p_1 are chosen so that the ratio $(\kappa_1 + \kappa_b)/\kappa_b$ does not exceed about two orders of magnitude. This prevents the elements F_{ii} from becoming too disparate. We note that the ith row of $\underset{\sim}{\mathbf{V}}$ is multiplied by F_{ii}, and if this value is too small, the ith row of $\underset{\sim}{\mathbf{V}}$ is essentially annihilated and there will be no possibility of adjusting the value of the ith cell. However, if the ratio is too small, the flexibility in the mapping will be restricted and this affects the convergence rate of the algorithm. In the limit that $\kappa_b \rightarrow \kappa_1$, the nonlinear mapping degenerates into a linear truncation and the inversion will not converge. However, between the above two extremes, there is a wide range of values for the ratio that can yield a good mapping. Based upon numerical experiments (Oldenburg and Li, 1994), we have chosen a value of 50.0 for this ratio for the examples throughout this paper.

DEPTH WEIGHTING

It is well known that static magnetic data have no inherent depth resolution. For instance, when minimizing $\|m\|_2^2 = \int m^2 \, dv$, structures tend to concentrate near the surface regardless of the true depth of the causative bodies. In terms of model construction, this is a direct manifestation of the nature of the kernels whose amplitudes rapidly diminish with depth. The tendency to put structure at the surface can be overcome by introducing a depth weighting to counteract this natural decay. Intuitively, a weighting that approximately compensates for the decay gives cells at different depths equal probability to enter into the solution with a nonzero susceptibility. Before proceeding with the details of the weighting function for magnetic inversion, we illustrate the necessity, and effectiveness, of such a weighting function using a simple 1-D problem.

Consider a set of data $\mathbf{d} = (d_1, \ldots, d_N)^T$ generated from the equation

$$d_i = \int_0^1 g_i(z) m(z) \, dz, \qquad i = 0, \ldots, N, \tag{12}$$

where the kernels are

$$g_i(z) = e^{-az} \cos(2\pi i z).$$

The decay factor e^{-az} causes the constructed model $m_c(z)$ to have structure concentrating toward the region of small z in the classic model construction that minimizes $\|m\|^2$, since the model will be a linear combination of the kernels, i.e.,

$$m_c(z) = \sum_{i=0}^{N} \alpha_i e^{-az} \cos(2\pi i z). \tag{13}$$

This is shown in Figure 4a and 4b for two different models. These models are constructed from five data ($i = 0, 4$) to which noise has been added. It is apparent that the constructed model is shifted toward small z where the amplitude of kernels is relatively large. One way to counteract the bias is to seek a solution in model space that is spanned by the nondecaying portion of the kernels, in this case just the cosine functions. The desired model would have the form

$$m_c(z) = \sum_{i=0}^{N} \tilde{\alpha}_i \cos(2\pi i z), \tag{14}$$

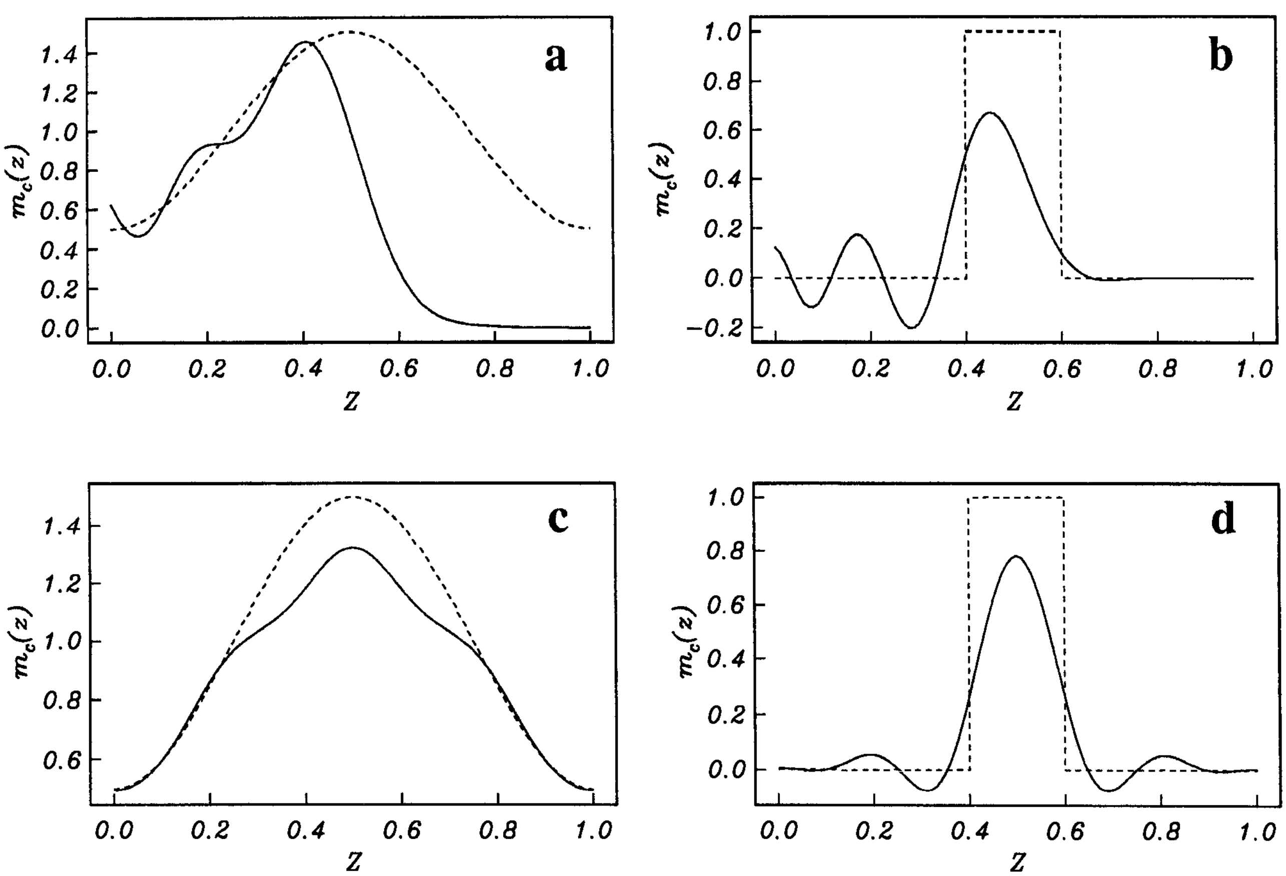

FIG. 4. A 1-D example showing the use of a weighting function in the inversion procedures to counteract the natural decay in the kernel function. In all panels the dashed line shows the true model. Panels (a) and (b) show, for the two different true models, respectively, the model constructed using the original kernel functions with the decaying factor e^{-az}. Notice the shift of the recovered model towards the small z region. Panels (c) and (d) show the weighted models recovered by applying a weighting function $w(z) = e^{-az/2}$. They are better representations of the true model.

where $\tilde{\alpha}_i$ are coefficients. Free from the influence from the decay factor, a model constructed from this set of basic functions should have a better chance of having significantly high values at depth.

We accomplish this by finding an appropriate weighting function $w(z)$. We first rewrite the data equation as

$$d_i = \int_0^1 \frac{g_i(z)}{w(z)} * w(z) m(z)\, dv \equiv \int_0^1 g_i^w(z) m^w(z)\, dv, \qquad (15)$$

where $g_i^w(z)$ are the weighted kernels and $m^w(z)$ is the weighted model. Then the inverse problem is solved by minimizing $\|m^w(z)\|^2$ and the solution is given by

$$m_c^w(z) = \sum_{i=0}^{N} \tilde{\alpha}_i g_i^w(z). \qquad (16)$$

Dividing $m_c^w(z)$ by the weighting function and substituting in $g_i^w(z)$ yields

$$m_c(z) = \sum_{i=0}^{N} \tilde{\alpha}_i \frac{g_i(z)}{w^2(z)} = \sum_{i=0}^{N} \tilde{\alpha}_i \frac{e^{-az}\cos(2\pi i z)}{w^2(z)}. \qquad (17)$$

This equation can be made identical to equation (14) by choosing $w(z) = e^{-az/2}$. Carrying out the weighted inversion for the above two data sets produces models shown in Figures 4c and 4d. They are much better representations of true models.

This methodology is then applied to the inversion of surface magnetic data by finding the appropriate weighting function that counteracts the depth decay of the data kernels. There is no distinct separable factor defining the decay in the kernel, therefore we resort to an empirical estimate. Since the decay rate depends upon the observation height as well as the size and aspect ratios of the cells making up the 3-D model, such estimates are expected to be problem dependent. Numerical experiments indicate that the function of the form $(z + z_0)^{-3}$ closely approximates the kernel's decay directly under the observation point, given a correctly chosen value of z_0. This is consistent with the fact that, to first order, a cubic-shaped cell acts like a dipole source whose magnetic field decays by inverse distance cubed. The value of z_0 can be obtained by matching the function $(z + z_0)^{-3}$ with the kernel function beneath the observation point. Thus, a reasonable candidate for the depth weighting function is given by

$$w(z) = \frac{1}{(z + z_0)^{3/2}}. \qquad (18)$$

The susceptibility model constructed by minimizing a model objective function consisting of only ϕ_{ms}, i.e.,

$$\phi_m(m) = \int_V (w(z)m(x, y, z))^2\, dv, \qquad (19)$$

subject to fitting the data should place the recovered anomaly at approximately the depth of the causative body. This hypothesis is tested by inverting surface data produced by a susceptible cubic body at three different depths. The cube is 200 m on a side. Data are calculated over a 21 × 21 grid of 50-m spacing in both directions, and 2% Gaussian noise is then added. The observation is assumed to be 1 m above the surface and the inducing field has $I = 75°, D = 25°$. The region directly beneath the data grid is taken as the model domain and discretized into 4000 cells (20 cells in each horizontal direction and 10 along depth) of 50 m on a side.

Given the stated data parameters and model discretization, the estimated value of z_0 in the depth weighting function is 25 m. Figure 5 shows the comparison of the kernel beneath a datum point and the function $w^2(z)$. This weighting function is used to invert surface data caused by the susceptible prism, and the results of minimizing ϕ_{ms} are shown in Figure 6. Each panel in the figure is the cross-section through the center of the model obtained by inverting the data set produced by a cube at a different depth. They are rather good recoveries in terms of source depth, which is indicated by the superimposed outline of the true body in each section.

In the above analysis we have established a practical way for estimating an appropriate depth weighting function that distributes the susceptibility more uniformly with depth. The weighting is valid when the model objective function consists only of ϕ_{ms}. In general, we like to include a penalty against roughness and thereby produce a model that is smooth. To incorporate the above weighting scheme in the spatial variations, we make the following argument. Since minimizing ϕ_{ms} tends to provide a reasonable depth distribution, we wish only to improve the model's smoothness while maintaining the depth characteristic. A conceptually consistent approach would be to apply the roughness measures to the weighted model. We form a generic model objective function

$$\phi_m(m) = \alpha_s \int_V w_s\{w(z)[m(\mathbf{r}) - m_0]\}^2\, dv$$
$$+ \alpha_x \int_V w_x \left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial x}\right\}^2 dv$$
$$+ \alpha_y \int_V w_y \left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial y}\right\}^2 dv$$

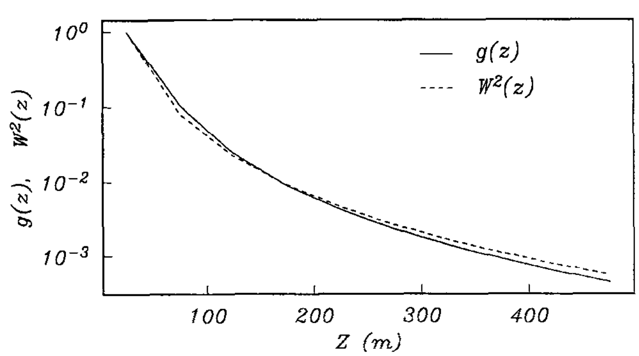

FIG. 5. Comparison of the kernel function (solid) directly beneath the observation point with the estimated curve (dashed) given by $w^2(z) = (z + z_0)^{-3}$ with $z_0 = 25$ m. The source cell is a cube of 50 m on a side. Here, z denotes the depth to the center of the cell. Both curves are normalized for comparison.

$$+ \alpha_z \int_V w_z \left\{ \frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial z} \right\}^2 dv, \qquad (20)$$

where the depth weighting is applied inside the derivatives of the roughness components and the reference model m_0 can be removed from any term if desired. This type of depth weighting has proven to work satisfactorily on a number of synthetic examples and is the default choice in our algorithm. The examples to be presented in the following sections all use this depth weighting function.

Before proceeding further, we remark that the above weighting represents only one possibility. One could potentially design a different weighting by incorporating the depth weighting in the usual 3-D weighting functions w_s, w_x, w_y, w_z. Such an approach applies the depth weighting outside the derivative operators directly. However, the decay rate of the depth weighting for each component will be different, and it is difficult to establish a consistent rule for the choice of the different weightings. In addition, the extra set of parameters required by such a weighting scheme introduces more subjectivity into the inversion process. We have not explored this approach in detail; however, it is observed that straightforward inclusion of the depth weighting derived above into the 3-D weighting function in the form of $\int_V w_z w^2(z)\{\partial[m(\mathbf{r}) - m_0]/\partial z\}^2\, dv$ can yield reasonable results.

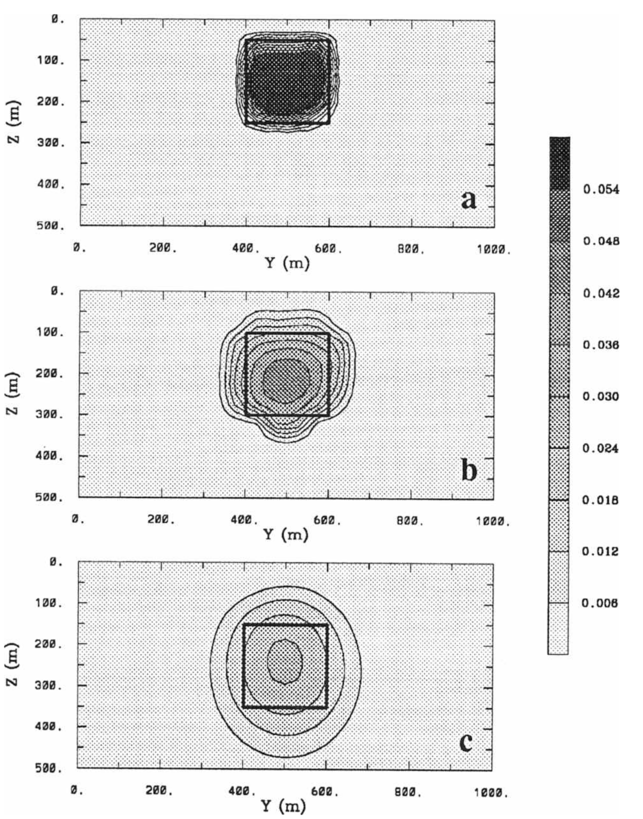

FIG. 6. Cross-sections through the center of the recovered model for a cube at a central depth of 150, 200, and 250 m. The cube is 200 m on a side. The inversion uses the weighting function derived from the kernel decay estimated in Figure 5. The true position of the cube is outlined in each cross-section. As the true source depth increases and, as a result, the high-frequency content in the data decreases, the recovered model becomes increasingly smooth and attains a smaller amplitude. However, the depth of the recovered model is close to the true value.

PRACTICAL ASPECTS OF DATA PREPARATION

The data used in the inversion are the residual data obtained by subtracting a regional field from the initial observation. The inversion algorithm has been developed under the assumptions that the surface magnetic anomaly is produced by the induced magnetization only and that there are no remanent magnetization or demagnetization effects present. Incorrect removal of regional field, or any deviation from the above assumptions, is expected to cause a deterioration in the inversion results. Furthermore, the susceptibility distribution is mathematically represented by a piece-wise constant function defined on a user-specified grid of cells. Magnetic sources, however, have a wide range of physical sizes. In some cases, source dimensions will be significantly smaller than the size of cells in the mathematical model. If measurements are taken close to such a source, the resulting anomaly will have a width that is significantly smaller than that produced by a single cell in the mathematical model and this may produce artifacts. We ameliorate this problem by inverting data that have been upward continued to a height approximately equal to the width of the surface cells in the model. We arrive at this conclusion from a numerical experiment. We first generate the magnetic field H_ℓ from a small localized surface source that is assumed to be a cube of width ℓ. At each height h above the surface, a one-parameter inverse problem is carried out to find a uniform susceptibility of a large surface cube that has a width of L and shares a common horizontal center with the small cube. If H_L is the field of the large cell that best reproduces H_ℓ then the misfit functional,

$$r(h) = \frac{\int_{\Delta s} (H_\ell - H_L)^2\, ds}{\int_{\Delta s} H_\ell^2\, ds}, \qquad (21)$$

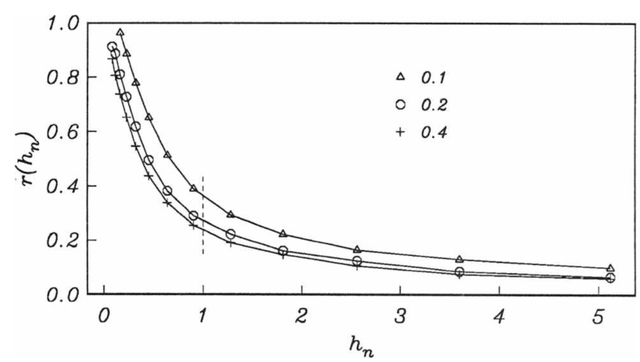

FIG. 7. The misfit between magnetic field as a result of a small cubic source and the field as a result of a larger cubic model cell having a best fitting susceptibility. The numbers indicate the ratio of the cell width. The misfit is plotted as a function of the observation height normalized by the width of the model cell. Note that the misfit decreases rapidly until the height is approximately equal to the width of the model cell, and that it changes slowly thereafter.

can be computed, where ΔS is the surface area of the data map. Figure 7 shows the misfit function $r(h)$ for trial values of $\ell/L = 0.1, 0.2, 0.4$. We note that $r(h)$ decreases rapidly until $h \approx L$, and that it changes slowly thereafter. Since the above misfit analysis is a worst case scenario because the contaminating body is located at the surface, the suggestion of upward continuing the data to a height approximately equal to the width of surface cells may be somewhat conservative, and inversionists may want to vary this. However, in many field surveys, magnetically susceptible small bodies exist close to the surface and hence upward continuing the data prior to inversion is prudent.

SYNTHETIC EXAMPLES

As the first example, we invert the total field anomaly data given in the Introduction. The model consists of a 3-D dipping slab buried in a nonsusceptible half-space (slab model). Figure 1 shows three slices through the slab model. The susceptibility of the slab is 0.06 (SI unit). Under an inducing field with a strength of 50 000 nT and a direction at $I = 75°$ and $D = 25°$, the slab model produces the surface total magnetic anomaly shown in Figure 2, which consists of 441 data over a 21×21 grid of 50-m spacing. The data have independent Gaussian noise added whose standard deviation is equal to 2% of the accurate datum magnitude plus 1 nT. We invert these 441 noise-contaminated data to recover the susceptibility of an earth model parametrized by 4000 cells of 50 m on a side (20 cells in each horizontal direction and 10 in depth).

The data are partitioned into 49 groups to provide 49 search vectors for the subspace algorithm. In addition, each component in the model objective function provides one basis vector, and a constant vector is included. For the depth weighting, the value of z_0 is estimated as 25 m. The additional 3-D weightings in the objective function are all set to unity. The reference susceptibility model is set to zero. For the nonlinear mapping, we choose $\kappa_b = 0.0002$ and $\kappa_1 = 0.01$.

First, we invert the data by minimizing an objective function composed only of the ϕ_{ms} and using $m = \kappa$ as the model parameter. A total of 51 subspace vectors are used at each iteration. The inversion reaches the expected misfit in 13 iterations but a few extra iterations are performed in an attempt to further reduce the value of the model objective function while keeping the misfit at the target value. By iteration 18, the objective function is decreasing by less than 1% per iteration, and the process is terminated. The constructed susceptibility model is shown in Figure 8 and can be compared with the true model in Figure 1. The tabular shape of the anomaly and its dipping structure are clear, and the depth extent is reasonably recovered. The amplitude of the recovered model is slightly higher than the true value, but the dip angle inferred from the recovered model is close to the true value. We point out that the model sections should be plotted using gray shading for each cell to reflect the piece-wise constant nature of the model. However, when the model has only a small number of cells in each spatial direction, the structural trends are more readily shown when contours are used. For this reason, we have contoured the model sections.

Next, the same data are inverted using a model objective function that includes penalty terms on spatial roughness, ϕ_{mv}. The depth weighting is applied to all terms, as in equation (20). The inversion uses 54 subspace vectors and achieves the expected misfit in 13 iterations. The recovered model is shown in Figure 9. It is smoother, has a slightly lower amplitude than the model in Figure 8, and it recovers the essential features of the true model such as the depth and dip angle.

It is observed, in this example and in other synthetic and field test examples, that minimizing either the first term in the model objective function in equation (20), ϕ_{ms}, or using all four terms, generates models that are reasonable representations of the true structure. In the absence of prior information, both models can provide useful information about the subsurface susceptibility distribution. However, the model minimizing ϕ_{ms} can be obtained at less computational cost. Further-

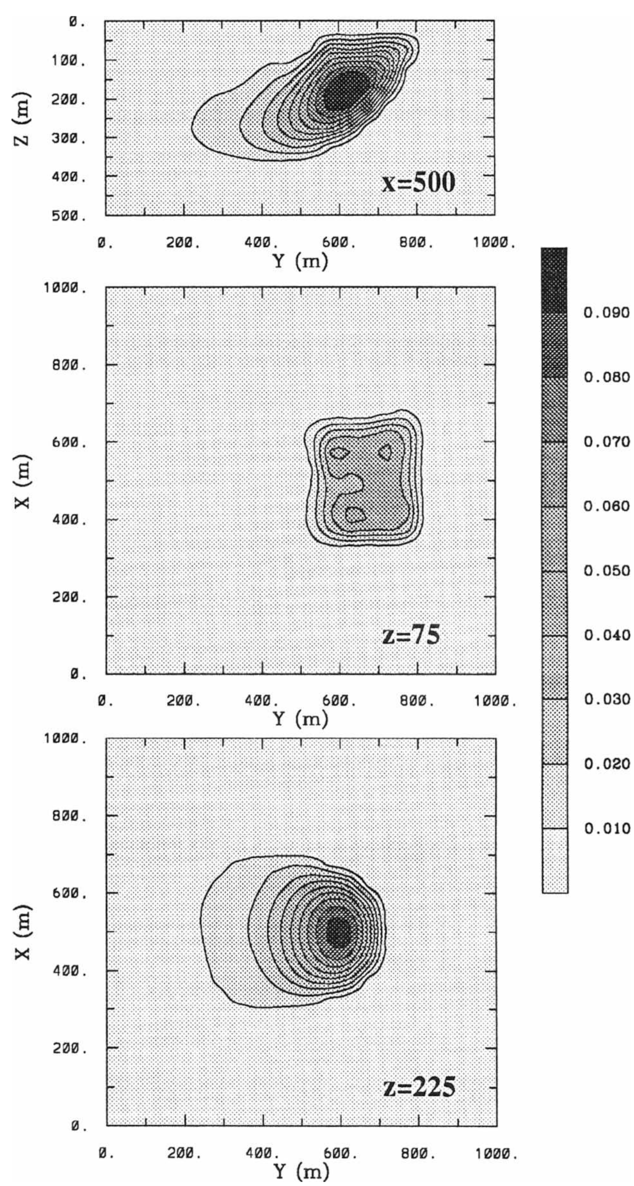

FIG. 8. Model obtained from inverting the data shown in Figure 2 by minimizing only ϕ_{ms}, which has the depth weighting applied. This is to be compared with the true model in Figure 1. The major features in the true model, such as dip angle and depth extent, are evident in the recovered model.

more, the depth weighting in this case is rather well supported by mathematical analysis whereas it is an argued extension for the three roughness components. Therefore, a reasonable approach to inverting field data might be a two-step process. The data can be inverted first by minimizing ϕ_{ms}, and the resultant model may be used in the interpretation as a preliminary result. If there are interesting features present and if one desires to refine the model by incorporating prior information to enhance or attenuate the structural complexity in different regions, a second inversion can be carried out using an objective function consisting of both ϕ_{ms} and ϕ_{mv}. The model obtained by minimizing ϕ_{ms} can then be used in this inversion as an initial model. The available prior information can be incorporated into the second inversion by forming a reference model and 3-D weighting functions, w_s, w_x, w_y, w_z.

We now invert the same data by using $m = \ln(\kappa)$ as the model. It is not possible to incorporate a zero susceptibility as the reference model, so we minimize an objective function consisting of ϕ_{mv} with the reference model removed. The same depth weighting is applied to all terms of ϕ_{mv}. Since $\kappa = e^m$, the positivity of the susceptibility is ensured without invoking the transformation of variables. The result is shown in Figure 10a. This is a cross-section at $x = 500$ m and plotted on a logarithmic scale in accordance with the model used in the inversion. The inverted susceptibility shows the presence of the dipping anomaly as a broad region of high susceptibility. However, the interpretation based upon such a model can be complicated by the variations of susceptibility that are small and have little effect on the surface data. We have replotted the cross-section on a linear scale in Figure 10b and the anomalous region is now delineated more clearly. Its top portion indicates the tabular body and defines the depth to the

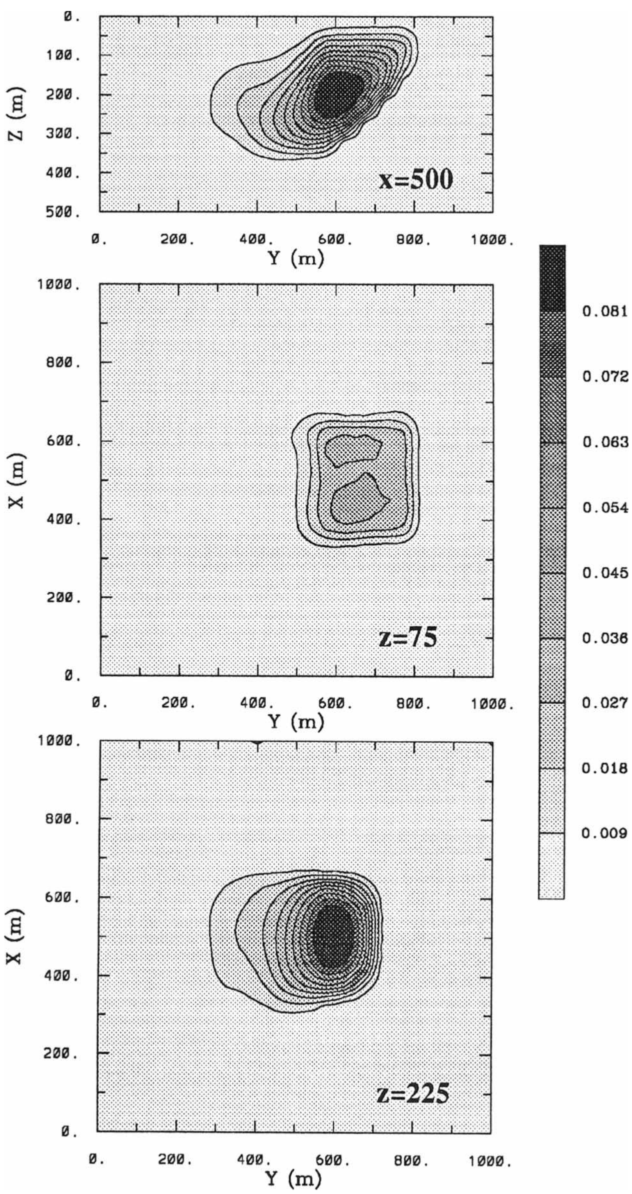

FIG. 9. The model derived from inverting the slab model data in Figure 2 by minimizing the model objective function having both ϕ_{ms} and ϕ_{mv}. The same depth weighting is used. This model appears to be smoother and has a smaller amplitude than that in Figure 8.

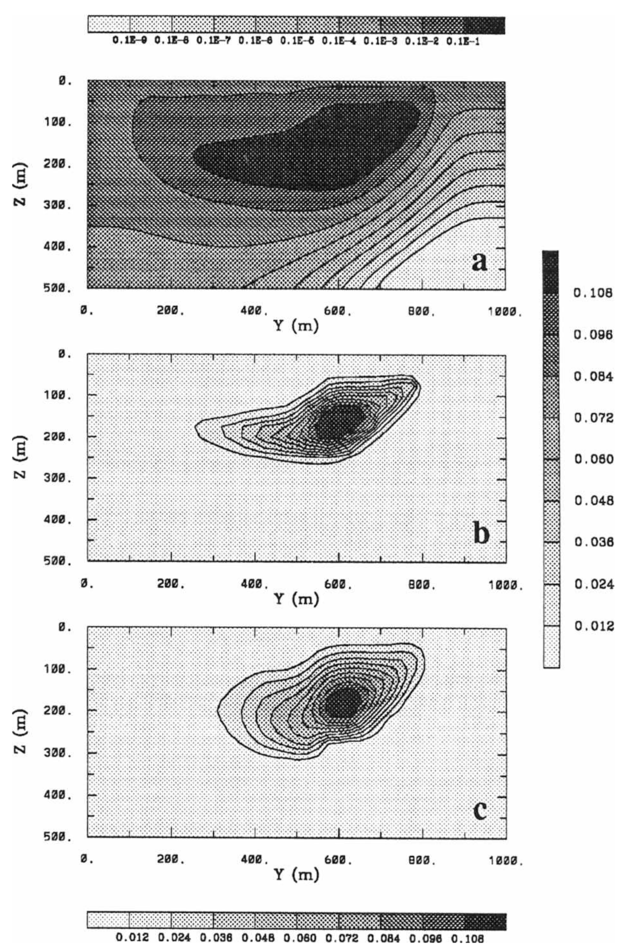

FIG. 10. The model obtained from inverting the data shown in Figure 2 by using $m = \ln(\kappa)$ as the model and minimizing ϕ_{mv} with the reference model removed. The inverted logarithmic susceptibility in cross-section at $x = 500$ m is shown in (a) and it is replotted on a linear scale in (b). As a comparison, the result obtained by using $m = \kappa$, and the same objective function is shown in (c).

top and dipping angle. The anomaly terminates at a shallower depth than the true model and has a nearly horizontal extension to the left. As an exact comparison, Figure 10c is the susceptibility model obtained by minimizing ϕ_{mv} but using $m = \kappa$ as model and invoking the positivity. This is a smoother model and exhibits more gradual changes in the susceptibility. It has a slightly deeper extent than the model in Figure 10b. With the exception of details toward the bottom, however, both models provide almost the same information about the anomalous susceptibility region. It might be concluded that inversion using either linear or logarithmic susceptibility is viable for practical applications. However, we note that the presentation in Figure 10b is inconsistent with the model used in the inversion. Since the inverted susceptibility is easier to interpret on a linear scale as demonstrated here, and since the magnetic data are linearly related to the susceptibility, we generally prefer to work with the susceptibility κ as the model in the inversion.

As the second example we invert the total field anomaly data produced by a slightly more complicated model and with two different inducing field directions. The true model is shown in Figure 11 in the same format as before. It is a dipping slab having its top and bottom portions offset to simulate the result of a normal faulting. The faulted slab strikes north. The data from this model, when the inducing field has a direction of $I = 45°$ and $D = 45°$, are shown in Figure 12. Again Gaussian noise has been added to the data. The inversion minimizes an objective function consisting of ϕ_{ms} and ϕ_{mv} that have the same depth weighting and nonlinear mapping as used to produce the results in Figure 9. Figure 13 displays the recovered model in three slices. It shows two distinct anomalous regions of susceptibility that correspond to those in the true model. The dipping structure is evident from the top block. On plan view, the strike direction and the strike length of the anomaly are also well recovered.

When the inducing field direction is $I = 0°$ and $D = 45°$, the surface anomaly with added Gaussian noise is that shown in Figure 14. Carrying out the inversion using an identical model objective function generates the model shown in Figure 15. It is similar to the model shown in Figure 13, which is recovered under an inducing field at 45° inclination. Again, the two separate blocks, the dipping direction, and the length and direction of the strike, are all reasonably recovered. This is a positive result in that, although the surface anomalies have very different expressions under different inducing field directions, the inversion algorithm is able to consistently recover the source structure. Moreover, the algorithm had no difficulty in inverting data generated from an inducing field having zero inclination; such data often pose problems in interpretations that include a reduction to pole.

We emphasize that positivity has played a pivotal role in all the inversions. Magnetic data generally have regions of nega-

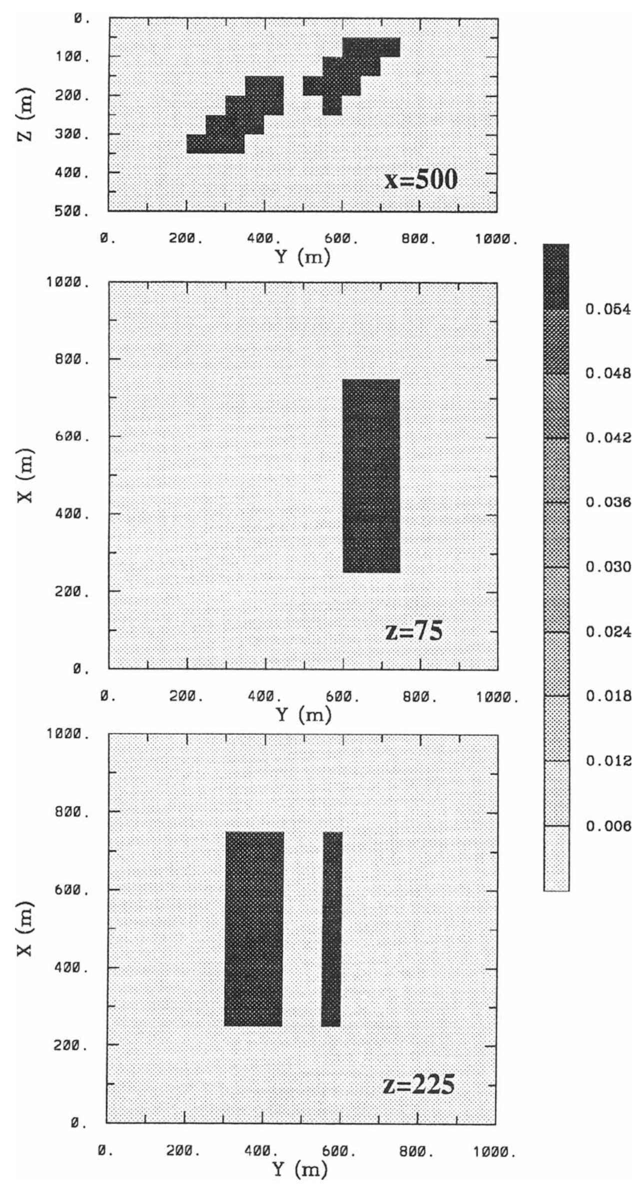

FIG. 11. The second synthetic test example. The top and bottom portions of the anomalous susceptibility are offset to simulate a norm fault structure. It also has a large strike length in the north direction.

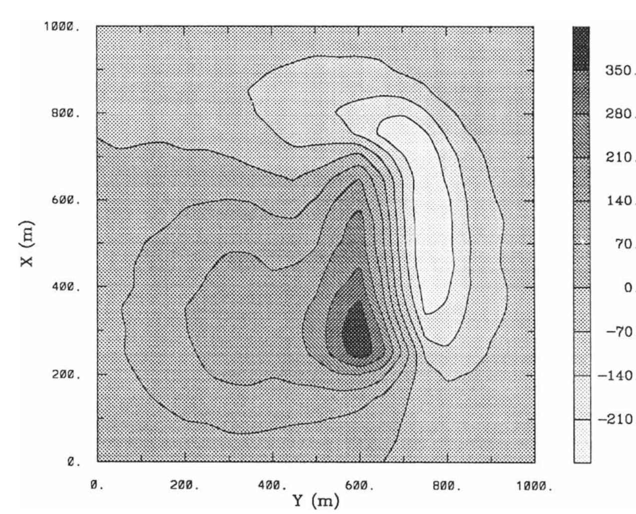

FIG. 12. The surface total field anomaly produced by the faulted slab in Figure 11, under an inducing field at $I = 45°$ and $D = 45°$. Uncorrelated Gaussian noise is again added to the data.

tive values that result from dipping bodies or inclined inducing field, or both. Without positivity, the constructed susceptibility is often negative and the dipping bodies appear more vertical. Recovery of correct dip and, to some extent, depth to the top of the anomalous body, are often the result of invoking positivity. Once the positivity is imposed, it is no longer true that an equivalent stratum that reproduces the data exists at any depth. Therefore, cells of anomalous susceptibility cannot be placed arbitrarily close to the surface, and no equivalent source can be constructed with negative susceptibilities. This restricts the class of admissible models and, consequently, reduces the nonuniqueness.

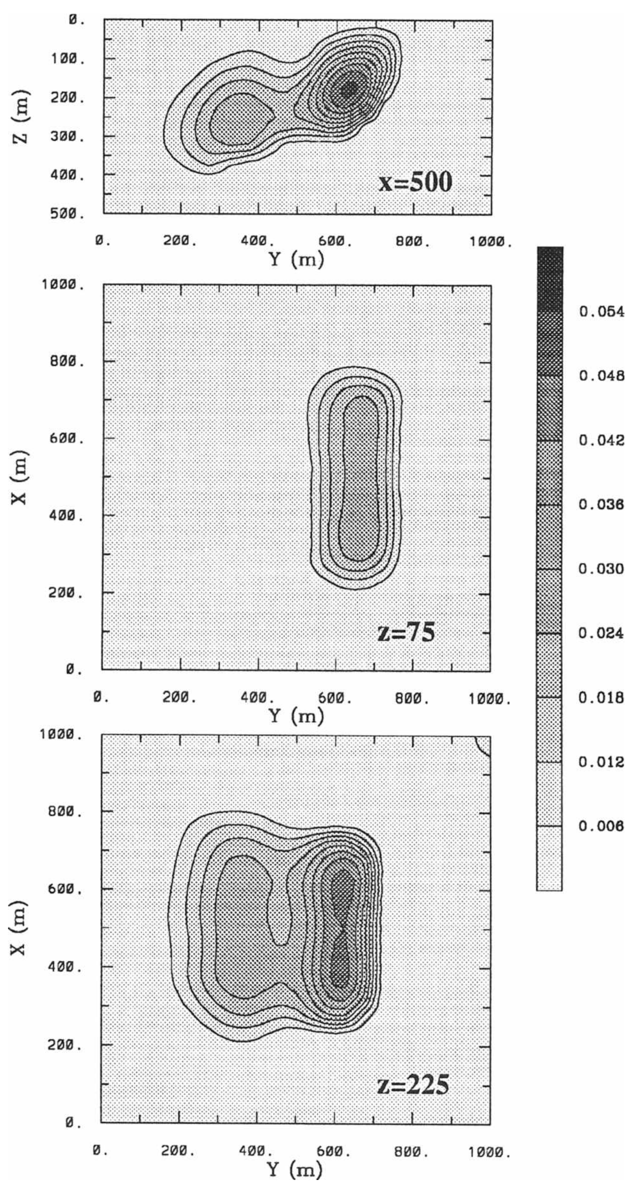

FIG. 13. The susceptibility model recovered from the data shown in Figure 12. It is seen that both the top and bottom block of the true model are recovered and the strike direction and length are well defined.

FIELD EXAMPLE

As the final example, we invert field data taken over a copper-gold porphyry deposit at Mt. Milligan in central British Columbia. The host rocks for the deposit are early Mesozoic volcanic and sedimentary rocks and contain intrusive monzonitic rocks that have accessory magnetite. Porphyry-style alteration and copper-gold mineralization are contemporaneous with the intrusive events. The copper and gold are known to be concentrated in the potassic alteration assemblage, which is mainly around the contact of the monzonite intrusions and may extend outward and into fractured volcanic rocks. Among other minerals, magnetite is one of the strong indicators of the potassic alteration. Ground magnetic data are acquired in the region at 12.5-m spacing along lines in the east direction and spaced 50 m apart. Our study of the data set has focused on a 1.2 km $\times$ 1 km area, which covers a large monzonite body known as the MBX stock and contains a reasonably isolated set of magnetic anomalies. Fairly detailed information about the geology is available through a major drilling program, but no susceptibility logs were available.

Magnetic data from a larger area were first upward continued to 20 m. A regional field was then defined and removed from the upward continued data. The continuation operation suppresses the noise in the data and also facilitates the discretization of the topographic surface for the model so that all observation points remain above the discretized surface. Although the original data were collected at 12.5-m spacing, we use the data at 25-m spacing. This yields 1029 data points at varying elevations. Figure 16 shows the data contoured according to their horizontal locations. The direction of the inducing field is $I = 75°$ and $D = 25.73°$. Several major magnetic highs are observed in the map. However, the influence of anomalies adjacent to the map is also visible along the edges. We choose a model domain that is horizontally larger than the data area, coincides at the top with the highest point on the topographic surface, and extends to 450-m depth. The model is discretized horizontally at a 25-m interval beneath the area of data. In the

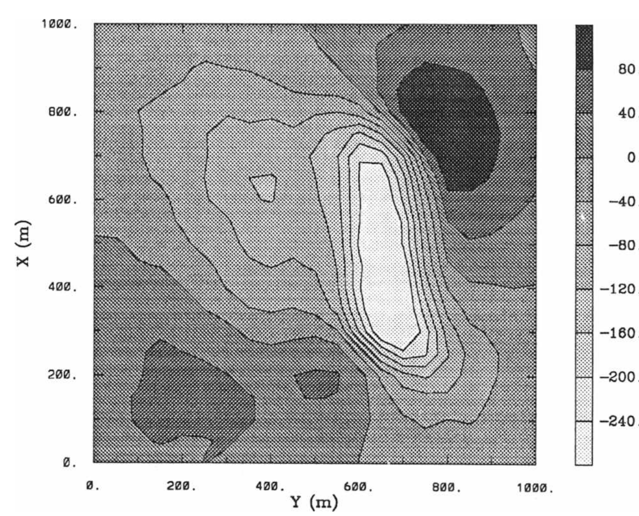

FIG. 14. The surface total field anomaly produced by the faulted slab in Figure 11 under an inducing field at $I = 0°$ and $D = 45°$. Uncorrelated Gaussian noise is added to the data.

vertical direction, the first 100 m is divided at a 12.5-m interval so that the surface can be adequately discretized onto the model mesh. Below the depth of 100 m, an interval of 25 m is used. This results in a mesh with $52 \times 44 \times 22$ cells. Once the mesh is defined, the topography is discretized onto it. The 43 428 cells below this surface define the susceptibility model, and the inverse problem is therefore formalized by inverting 1029 data to recover the susceptibilities in those cells. The depth weighting is referenced to the top of the model domain. Each datum is assumed to have an error whose standard deviation is equal to 5% of its magnitude plus 10 nT. The error estimate includes not only the repeatability of the instrument reading but also the geological noise and errors introduced by the inaccurate recording position and by separating the anomalous field from the initial total field measurements. One hundred subspace vectors generated by dividing the data map into small subareas are used in the inversion. We use a nonlinear mapping with $\kappa_b = 0.0002$ and $\kappa_1 = 0.02$. The recovered model is shown in Figure 17 as one plan-section and three cross-sections. From the plan-section, two concentrated susceptibility highs are observed in the central region. Surrounding them are three linear anomalies trending northeast. In the cross-sections, the major anomalies are seen at moderate depths but there is considerable variation in the depth to the top. There are also smaller anomalies extending to the surface. In general, there are more detailed structures near the surface and the model becomes increasingly smooth at greater depths. As required by the objective function, there is no excessive structure associated with each unit of high susceptibility region. Comparison with drill logs indicates that the recovered magnetic susceptibility highs are mostly associated with the monzonite intrusions and with faults or fracture zones. Figure 18 compares the recovered susceptibility model with the geology (Cam DeLong, personal communication) in the cross-section at $x = 600$ m. The large susceptibility high is spatially well-correlated with the MBX stock and reflects the initial magnetite content in the intrusion. Two smaller susceptibility highs are present east of the stock. The high at $y = 650$ m coincides with the boundary of stock and porous trachytic units while the high at $y = 900$ m coincides with the upper portion of the Rainbow dyke. These are locations of the most intensive potassic alterations and the susceptibility highs are indicative of the magnetite produced by the alteration process. Over all, this is a rather encouraging result.

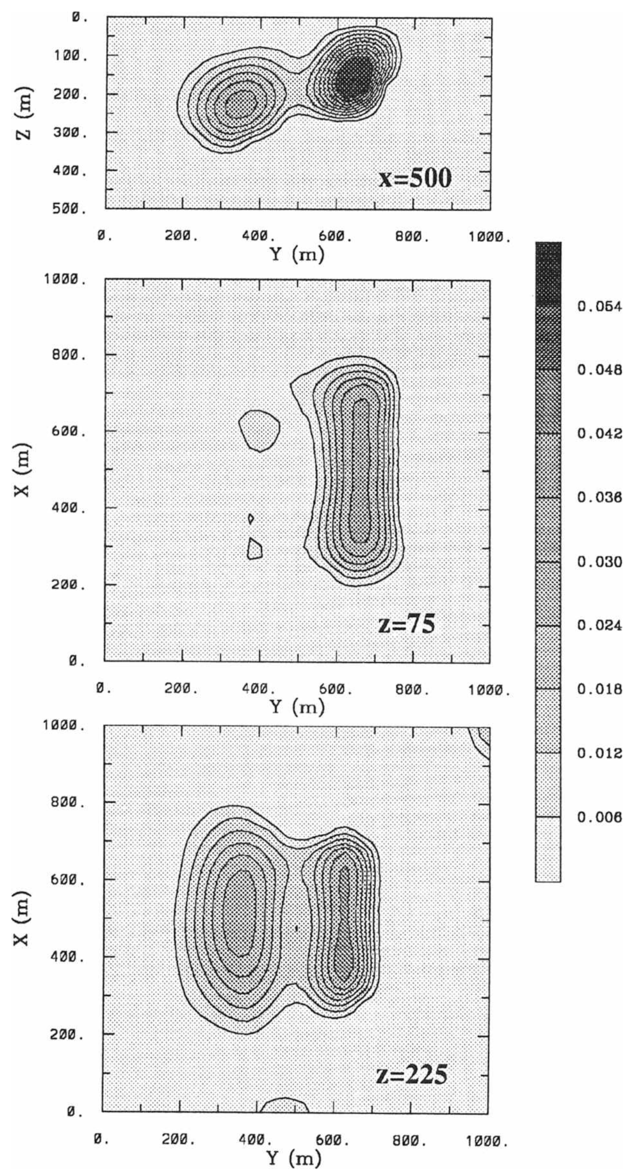

FIG. 15. The susceptibility model recovered from the data shown in Figure 14. This model is similar to that shown in Figure 13.

CONCLUSION

We have developed an algorithm to invert surface magnetic data for general 3-D susceptibility distributions. Although we have illustrated the algorithm using examples on the scale pertinent to mining applications, the method is general and applicable to problems on different scales ranging from environmental to regional investigations. To overcome the inherent nonuniqueness, we obtain the solution by minimizing a

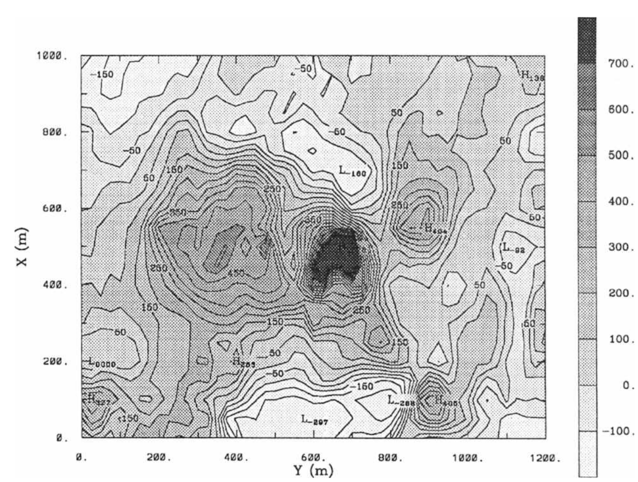

FIG. 16. The extracted total field anomaly from ground magnetic data at Mt. Milligan Copper-gold porphyry deposit. The data are contoured according to their horizontal locations in this map, although they are at different elevations.

specific objective function of the model. Our model objective function has the ability to incorporate prior information into the inversion via a reference model and 3-D weighting functions. A crucial feature of the objective function is a depth weighting function that counteracts the natural decay of the kernel functions. The parameters of the depth weighting depend upon the discretization of the model but are easily calculated. The minimization is carried out using a subspace technique that reduces the computational effort and allows the positivity constraint of susceptibility to be incorporated. Both susceptibility and logarithmic susceptibility can potentially be used as the model in the inversion. Since the data are linearly related to susceptibility, and since usually absolute values of susceptibility are required for interpretation rather than relative values, especially in regions of very low susceptibility, we have generally chosen to work with susceptibility. To suppress the noise from small magnetic bodies near the surface, we recommend in general that the data be upward continued to a height comparable with the width of the surface cell before inversion.

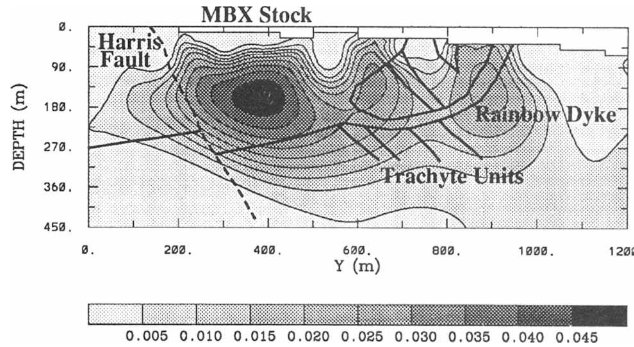

FIG. 18. Comparison of the recovered susceptibility model in a cross-section ($x = 600$) with the geology for the Mt. Milligan deposit. The susceptibility high within the MBX stock reflects the initial magnetite in the intrusive while the susceptibility highs near the Rainbow dyke are related to the magnetite produced by potassic alteration.

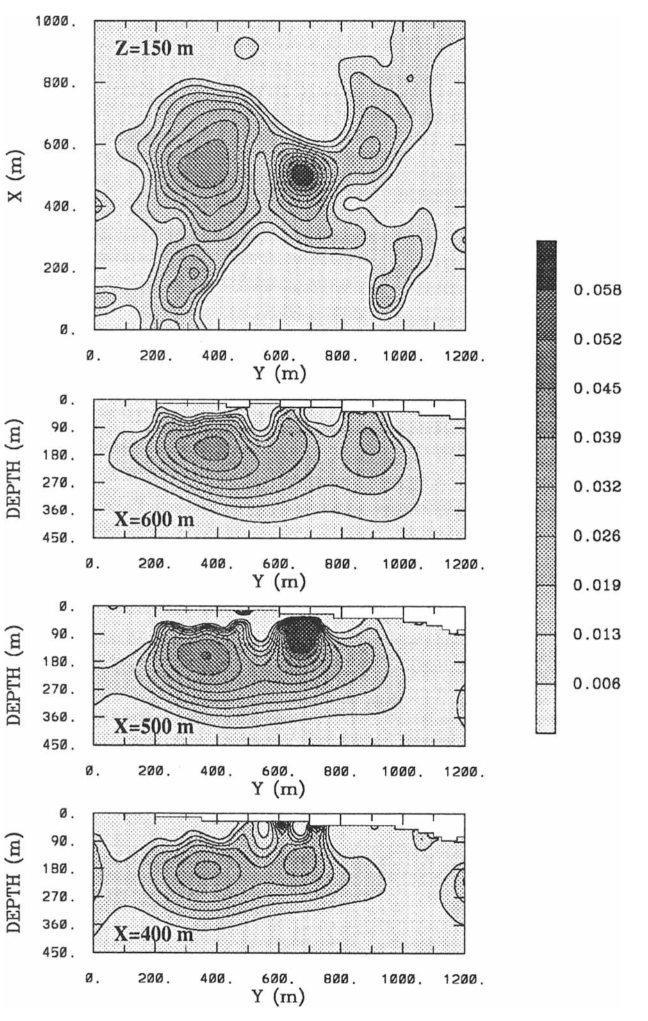

FIG. 17. The recovered susceptibility model shown in one plan-section and three cross-sections. The plan-section is at the depth of 150 m and the three cross-sections are at $x = 600$, 500, and 400 m, respectively.

Applications of our inversion to synthetic data sets have produced models representative of the true structures and demonstrated the ability of the algorithm to construct consistent models at different magnetic latitudes. Inversion of field data has produced a susceptibility model that is consistent with the known geology and mineralization information. These results represent an encouraging conclusion: although the inversion of magnetic data seems impossibly nonunique when one has a large number of cells, inversions using a properly designed model objective function can produce susceptibility distributions that yield meaningful geologic information.

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APPENDIX

MODEL OBJECTIVE FUNCTION

Our inversion method uses a model objective function of the form

$$\phi_m(m) = \alpha_s \int_V w_s\{w(z)[m(\mathbf{r}) - m_0]\}^2\,dv$$

$$+ \alpha_x \int_V w_x \left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial x}\right\}^2 dv$$

$$+ \alpha_y \int_V w_y \left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial y}\right\}^2 dv$$

$$+ \alpha_z \int_V w_z \left\{\frac{\partial w(z)[m(\mathbf{r}) - m_0]}{\partial z}\right\}^2 dv. \quad \text{(A-1)}$$

The numerical evaluation of this functional is carried out by introducing the model mesh and evaluating all terms using a finite-difference approximation. The discretized model objective function has the form

$$\phi_m(\mathbf{m}) = (\mathbf{m} - \mathbf{m}_0)^T(\underset{\sim}{\mathbf{W}}_s^T\underset{\sim}{\mathbf{W}}_s + \underset{\sim}{\mathbf{W}}_x^T\underset{\sim}{\mathbf{W}}_x + \underset{\sim}{\mathbf{W}}_y^T\underset{\sim}{\mathbf{W}}_y$$

$$+ \underset{\sim}{\mathbf{W}}_z^T\underset{\sim}{\mathbf{W}}_z)(\mathbf{m} - \mathbf{m}_0)$$

$$\equiv (\mathbf{m} - \mathbf{m}_0)^T\underset{\sim}{\mathbf{W}}_m^T\underset{\sim}{\mathbf{W}}_m(\mathbf{m} - \mathbf{m}_0). \quad \text{(A-2)}$$

Each component matrix can be written as the product of three individual matrices and one coefficient. That is,

$$\underset{\sim}{\mathbf{W}}_i = \alpha_i \underset{\sim}{\mathbf{S}}_i \underset{\sim}{\mathbf{D}}_i \underset{\sim}{\mathbf{Z}}, \qquad i = s, x, y, z, \quad \text{(A-3)}$$

where $\underset{\sim}{\mathbf{S}}_i$ are diagonal matrices representing the spatially dependent 3-D weighting functions, $\underset{\sim}{\mathbf{D}}_i$ are the finite-difference operators for each component, and $\underset{\sim}{\mathbf{Z}}$ is a diagonal matrix representing the discretized form of depth weighting function $w(z)$.

The elements of $\underset{\sim}{\mathbf{S}}_i$ are given by $\sqrt{w_i}$. They are defined over each cell for $\underset{\sim}{\mathbf{S}}_s$, and over each interface between adjacent cells in the respective directions for $\underset{\sim}{\mathbf{S}}_x$, $\underset{\sim}{\mathbf{S}}_y$, and $\underset{\sim}{\mathbf{S}}_z$. $\underset{\sim}{\mathbf{D}}_s$ has elements $\sqrt{\Delta x \Delta y \Delta z}$ on its diagonal, where Δx, Δy, and Δz are the cell width. The matrix $\underset{\sim}{\mathbf{D}}_x$ has two elements $\pm\sqrt{\Delta y \Delta z/\delta x}$ in each row, where δx is the distance between the centers of cells adjacent in the x-direction. Similarly, $\underset{\sim}{\mathbf{D}}_y$ and $\underset{\sim}{\mathbf{D}}_z$ have elements $\pm\sqrt{\Delta x \Delta z/\delta y}$ and $\pm\sqrt{\Delta x \Delta y/\delta z}$, respectively, where δy and δz are the distances between centers of adjacent cells in the y- and z-directions. Once the mesh is defined and all weighting functions, w_s, w_x, w_y, w_z, and $w(z)$ are chosen, equation (A-3) is evaluated straightforwardly and $\underset{\sim}{\mathbf{W}}_m^T\underset{\sim}{\mathbf{W}}_m$ is formed.